

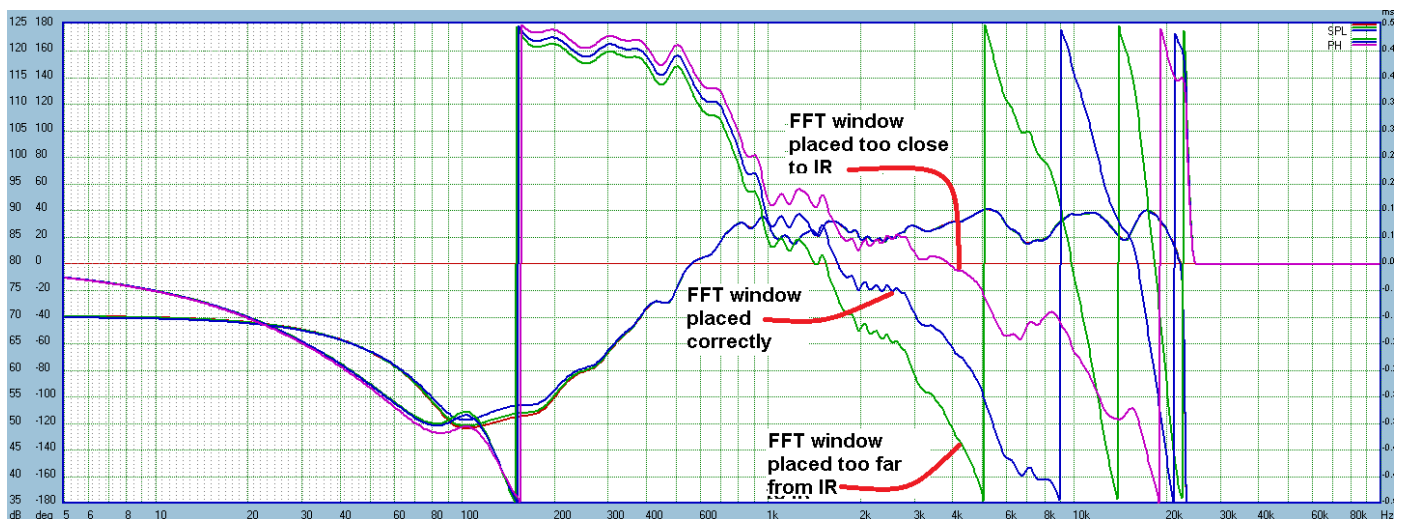
## Automated Extraction of Minimum-Phase Phase response

By Bohdan Raczynski – July 2021

When measuring loudspeaker phase response, there is a strong possibility, that the phase will be measured erroneously. There are three distinct possibilities, typically obtained from the measurement:

1. The FFT window is placed correctly – this will result in subtracting the exact amount of time-of-flight and the “leftover” will result in **exact minimum-phase phase response**. This is the desirable option.
2. The FFT window is placed too close – this will result in erroneous phase response, which is too flat. The “leftover”, which was supposed to represent the minimum-phase phase response is too shallow. Too much of the time-of-flight has been removed.
3. The FFT window is placed too far from the impulse response – this will result in the “leftover”, which was supposed to represent the minimum-phase phase response, still containing small residual time delay.

The three options listed above are depicted on a measurement example below.



The previous paper introduced **Inverse Hilbert-Bode Transform (IHBT)** and provided extensive description of it's primary application. A manual method of extracting exact “minimum-phase” phase response from measured SPL curve was also given and explained in details.

URL: [https://www.bodziosoftware.com.au/IHBT\\_White\\_Paper.pdf](https://www.bodziosoftware.com.au/IHBT_White_Paper.pdf)

The method required flipping between MLS measurement system screens and IHBT dialogue box several times (up to 20-30 times), before it produced the expected minimum-phase response.

This paper introduces automated method of extracting exact “minimum-phase” phase response from measured SPL curve. As the loudspeaker driver is a minimum-phase device, it's transfer function and phase response are mathematically locked together. Knowing SPL transfer function we can extract phase response from it. And conversely, knowing phase response, we can re-construct SPL transfer function from it. The re-constructed SPL transfer function will be identical to the original SPL transfer function – but **ONLY** if the supplied phase response was the “minimum-phase” phase response.

We can therefore supply a number of phase responses to the IHBT algorithm and compare resulting re-constructed SPL transfer functions with the originally measured SPL. The comparison is done by calculating squared error between the original and re-constructed SPL transfer functions.

Typical MLS and ESS measurement systems will both generate Impulse Response (IR) of the loudspeaker under test. The next immediate step is to apply an FFT windowing process to the recovered IR and then perform FFT function on the windowed IR. The outcome of this technique is SPL and phase response of a loudspeaker. The location of the start of the FFT window will determine the resulting phase response. The start of the FFT window can only be anchored at discrete time samples determined by signal sampling frequency. Typically, sampling will be set at 48kHz or 96kHz. For example, 48kHz sampling frequency will result in discrete time samples spaced 20.833usec apart. Therefore, corresponding phase response can only be obtained at those time samples.

Yes, our measurement system is quite coarse. The problem with this approach is, that the actual minimum-phase response may be corresponding to the FFT window placed somewhere between the discrete time samples. To overcome this shortcoming, a small positive or negative time delay can be added to the calculated phase response, and the resulting phase response is then supplied to the IHBT algorithm.

Clearly, the automated method described in this paper is a **two-stage process**. In Stage I, the FFT window is placed at 10 arbitrary, but consecutive time samples, and resulting phase responses are supplied to the IHBT algorithm. This way, we obtain 10 SPL responses and one of them will be the best match with the originally measured SPL. The time sample corresponding to this SPL curve is the starting point of Stage II.

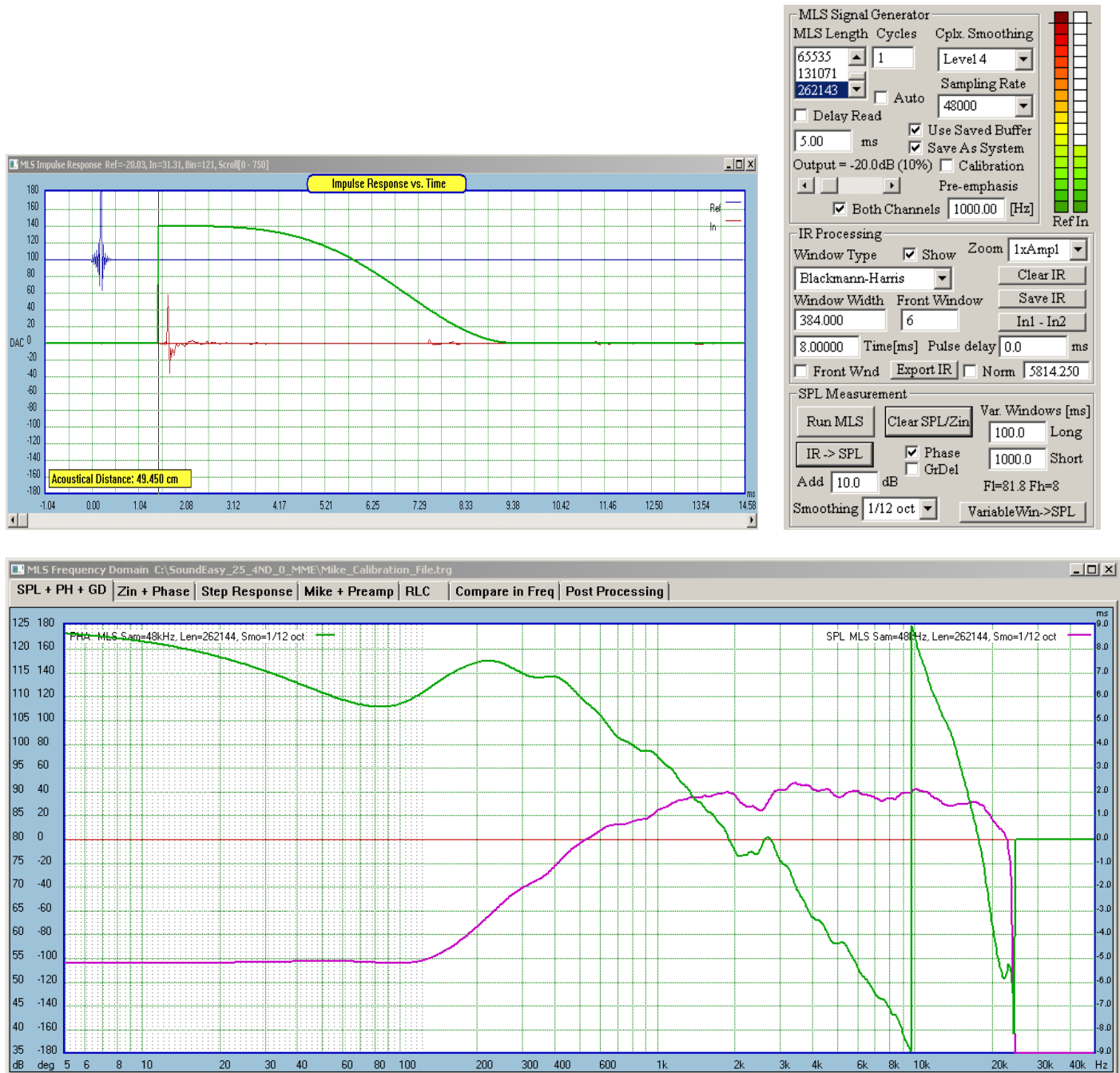
In Stage II, a small positive or negative time delay is added to the calculated phase response, and the resulting phase response is then supplied to the IHBT algorithm. Practically, the algorithm adds or subtracts ten 2usec time delays so that the new phase is recalculated across 20usec to the left and 20usec to the right of the sampling time determined from Stage I.

There are many options of selecting parameters driving the whole process, but for the sake of clarity of this presentation, they will not be discussed here.

We are now in the position to present two examples of measured loudspeakers a 12" guitar driver in a vented box and a D25 tweeter. The method described above will be used to extract the exact minimum-phase phase response in both cases.

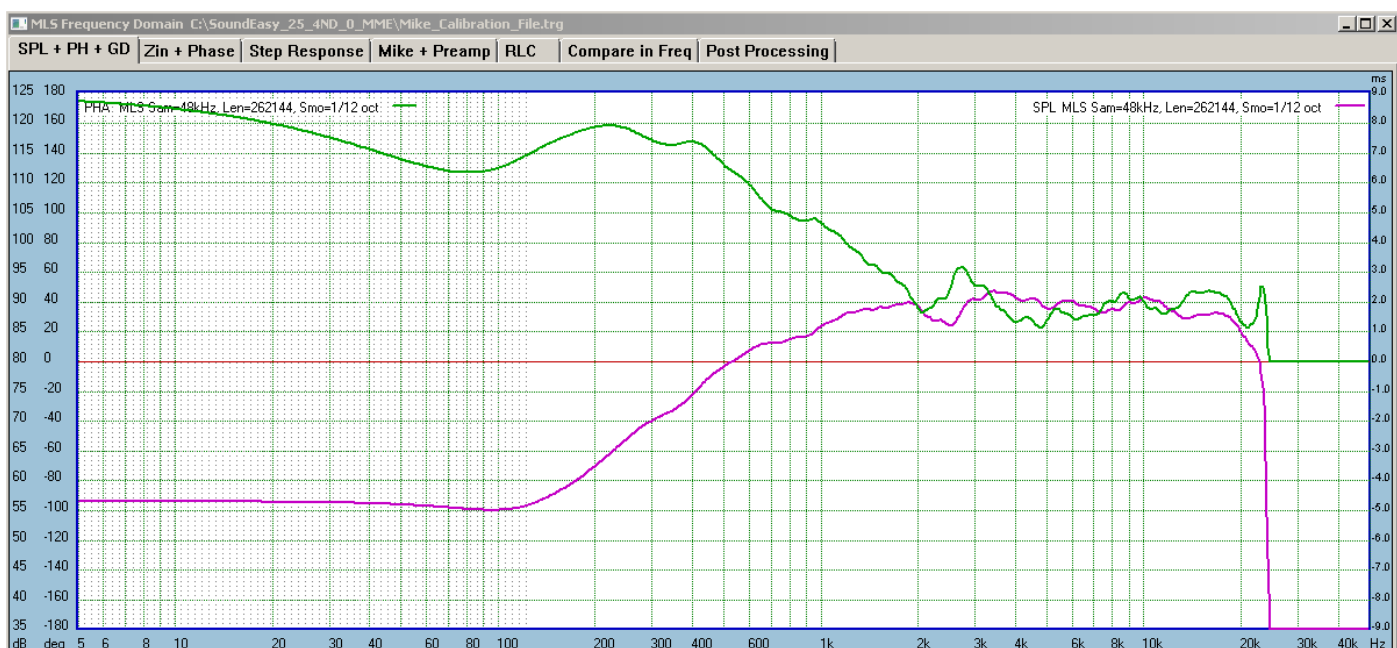
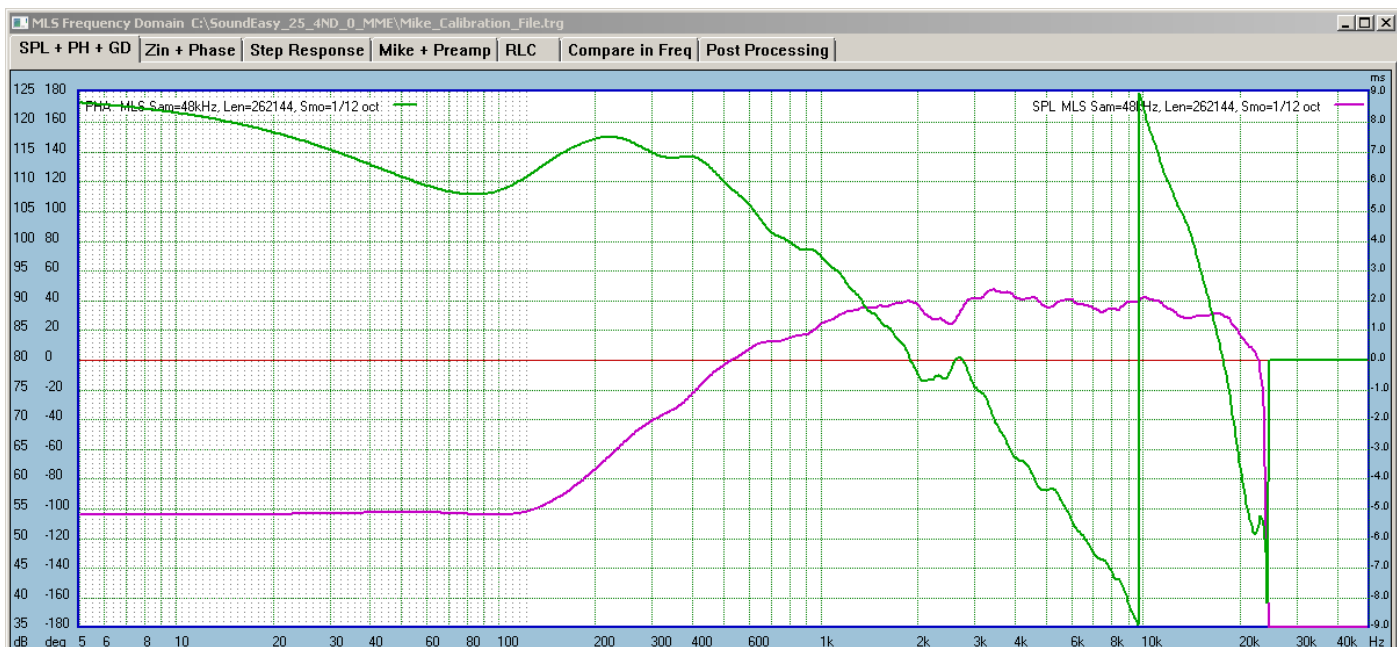
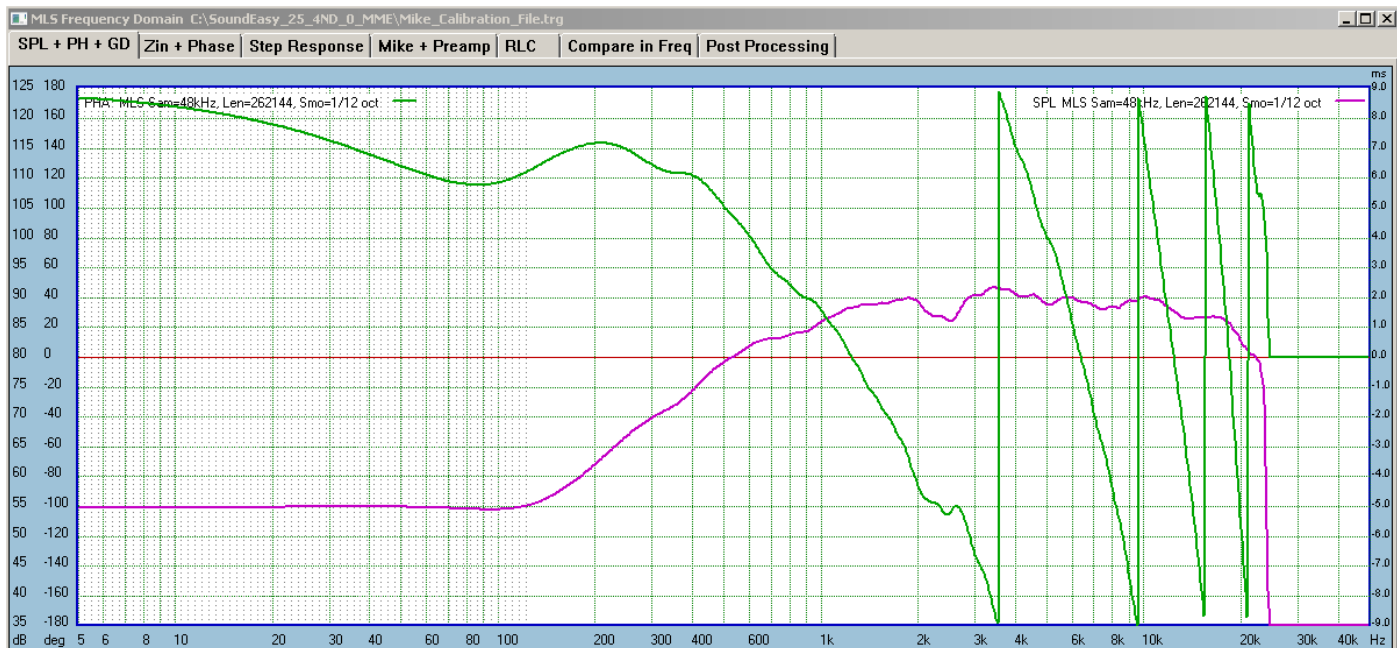
## Tweeter Driver Example

From the phase extraction point of view, tweeter drivers are notoriously difficult to deal with, because there is no SPL/Phase data above the Nyquist Frequency (half the sampling frequency of 48kHz). An example of tweeter driver measured up to 24kHz is shown below. Phase response was derived with the FFT window placed at time sample “Bin 121”.



Now, we are grappling with the same question – **how to determine the correct FFT window placement?**. Shown below are three examples of FFT window placement, and corresponding phase response (green curves).

On the top picture, the FFT was placed at Bin 116, resulting in perhaps too many +180/-180 deg phase transitions. On the middle picture, the FFT window start was placed at Bin 121. Here, we only have one phase transition – but is this correct?. On the bottom picture, the FFT window was placed at Bin 124, resulting in phase response having no transitions at all.

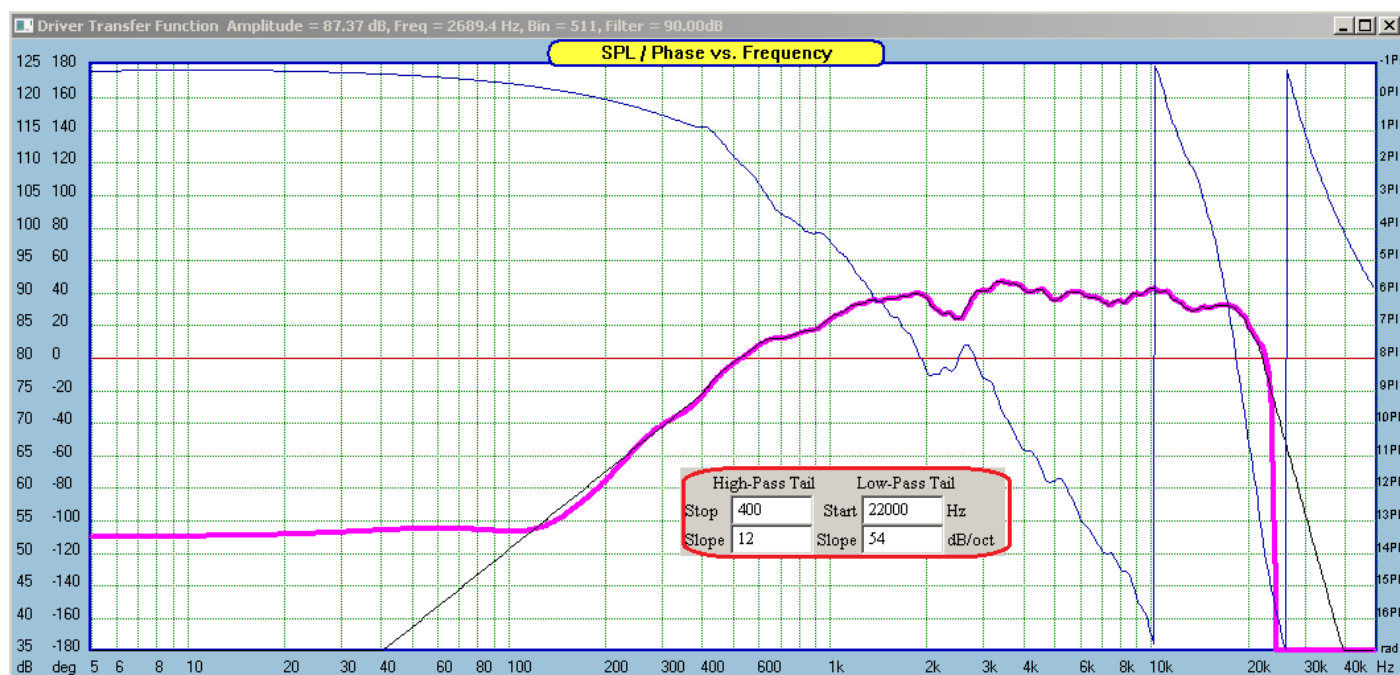


As explained in Part I article, the IHBT requires phase slopes to be attached to the phase response. There are several reasons for this.

1. At high frequency range, the measured phase response is limited to 5Hz – ½ sampling frequency and does not extend beyond, to the screen limit.
2. At low frequency range, the SPL is heavily distorted by the effect of FFT windowing.
3. At both: high and low frequency ranges the measured SPL and phase responses accuracy is limited by the measurement system dynamic range. There are sound reflections, and background noises, that typically produce garbage SPL/Phase responses at the frequency fringes. This needs to be removed.

Therefore, attaching the asymptotic “phase tails” would clean up the measurements perfectly and would enable the SPL/Phase to be calculated over the whole frequency range of 5Hz-50kHz. As it was explained in Part I, the phase slopes are attached via SPL slopes mechanism – so, that the process looks more familiar to the user.

Most importantly, the SPL calculated via IHBT is not sensitive to the attached slopes, as it is calculated only between it's designated frequency points, that are different from the attachment points of the phase slopes. In our example, the phase slopes are attached at 400Hz (+12dB equivalent) and 22kHz (-54dB equivalent). While the IHBT is calculated between 1000Hz and 21kHz.





## Step 1 – 10 Coarse FFT window placements

As explained before, in Stage I, the FFT window is placed at 10 arbitrary, but consecutive time samples, and resulting phase responses are supplied to the IHBT algorithm. This way, we obtain 10 SPL responses and one of them will be the best match with the originally measured SPL.

**Driver Parameter Editor**

**T/S Editor | Amplitude Model | Impedance Model | Hilbert-Bode Transform | Inverse H-B Transform**

High-Pass PHASE Tail Equivalent To SPL    Low-Pass PHASE Tail Equivalent To SPL    3. Inverse HBT Optimizer

Stop  Hz    Start  Hz

Slope  dB/oct    Slope  dB/oct

1. Phase Reference ----- 2. Inverse HBT ----- 2.1 Show -----

   ☐ Unwrapped Phase

   ☒ Measured Phase

   ☒ HBT Phase

   ☒ Search Run

☐ Show Minimum-Phase Guiding Filter

Run 0, Bin 116, Error 9117.56, Gain -15.41, Angle 87.80, Del 0.000

Run 1, Bin 117, Error 6055.45, Gain -5.01, Angle 104.66, Del 0.000

Run 2, Bin 118, Error 3611.22, Gain 5.29, Angle 121.87, Del 0.000

Run 3, Bin 119, Error 1799.74, Gain 15.73, Angle 138.73, Del 0.000

Run 4, Bin 120, Error 611.22, Gain 25.49, Angle 155.65, Del 0.000

Run 5, Bin 121, Error 61.57, Gain 36.57, Angle 172.89, Del 0.000

Run 6, Bin 122, Error 137.13, Gain 46.15, Angle 190.00, Del 0.000

Run 7, Bin 123, Error 833.52, Gain 57.64, Angle 207.36, Del 0.000

Run 8, Bin 124, Error 2181.69, Gain 66.14, Angle 223.76, Del 0.000

Run 9, Bin 125, Error 4094.00, Gain 78.93, Angle 241.75, Del 0.000

   From [Hz]     to [Hz]

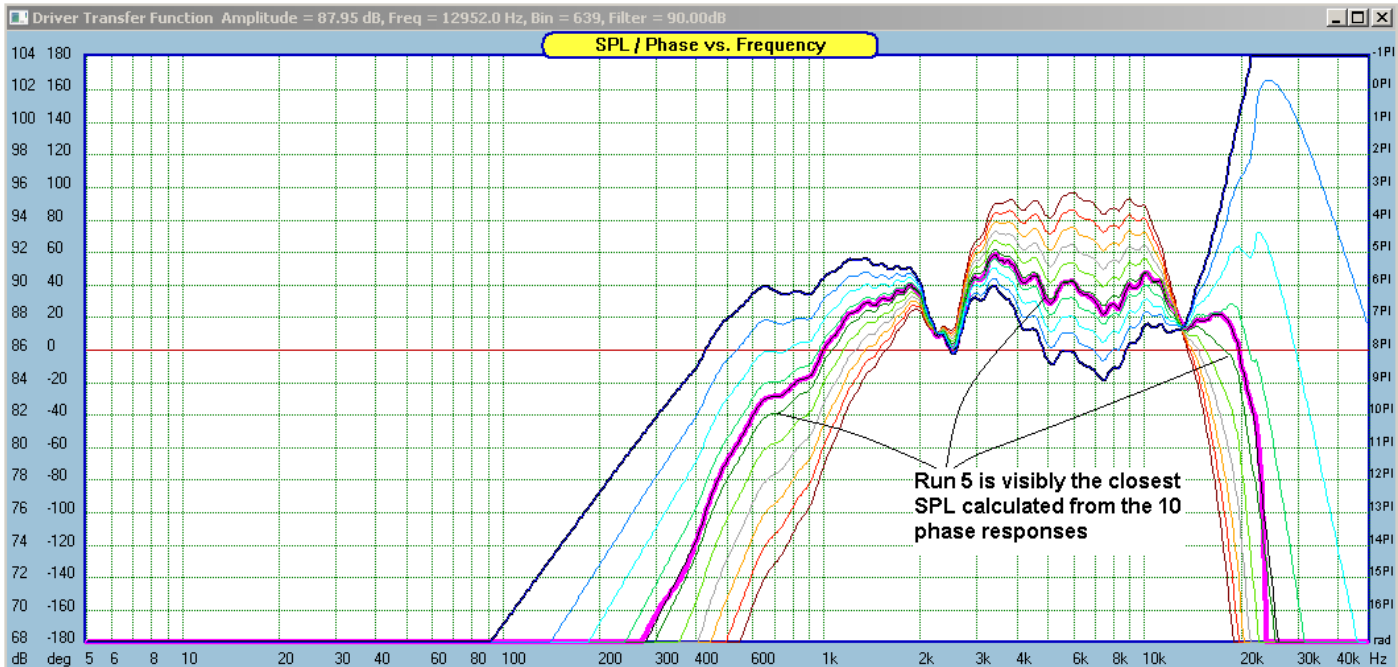
Count = 19    Gain = 78.93    78.93

Error = 4093.999    Angle = 241.75    241.75

Run = 125

Shown below, are the 10 SPL curves re-created by the IHBT algorithm. Please note, that the vertical scale resolution has been enhanced from 5dB to 2dB in order to increase visual separation between SPL curves. The pink curve is the originally measured SPL, and it is our “reference” SPL curve.

Next, the process automatically searched for the minimum “Error” value and selected Run 5 (or time sample Bin 121) as the starting point for the Stage II.



Stage II is started automatically.

## Step 2 – 20 Fine FFT (positive and negative) window delays

In Stage II, the algorithm adds or subtracts ten 2usec time delays so that the new phase is recalculated across 20usec to the left and 20usec to the right of the sampling time determined from Stage I.

**Driver Parameter Editor**

**T/S Editor | Amplitude Model | Impedance Model | Hilbert-Bode Transform | Inverse H-B Transform**

High-Pass PHASE Tail Equivalent To SPL    Low-Pass PHASE Tail Equivalent To SPL    3. Inverse HBT Optimizer

Stop  Hz    Start  Hz

Slope  dB/oct    Slope  dB/oct

1. Phase Reference    2. Inverse HBT    2.1 Show

   ☐ Unwrapped Phase

   ☒ Measured Phase

   ☐ HBT Phase

   ☒ Search Run

☐ Show Minimum-Phase Guiding Filter

Start Filter    From [Hz]     To [Hz]

Count = 16    Gain = 45.77    45.77

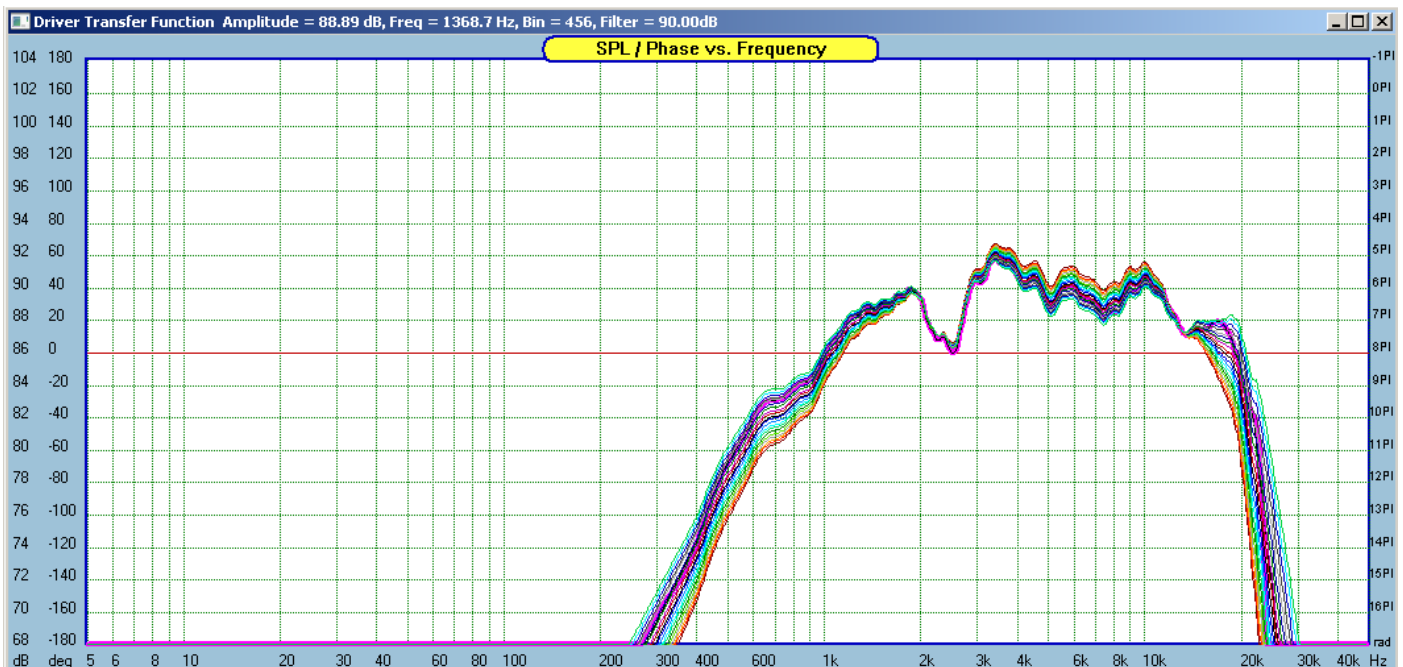
Error = 90.780    Angle = 188.03    188.03

Run = 121    **Bin 121, Error 15.163, Del 0.008**

Run	Bin	Error	Del
Run 0	Bin 121	Error 589.75	Del -0.020
Run 1	Bin 121	Error 510.41	Del -0.018
Run 2	Bin 121	Error 436.86	Del -0.016
Run 3	Bin 121	Error 369.37	Del -0.014
Run 4	Bin 121	Error 307.78	Del -0.012
Run 5	Bin 121	Error 252.05	Del -0.010
Run 6	Bin 121	Error 202.47	Del -0.008
Run 7	Bin 121	Error 158.35	Del -0.006
Run 8	Bin 121	Error 120.20	Del -0.004
Run 9	Bin 121	Error 88.05	Del -0.002
Run 10	Bin 121	Error 61.57	Del 0.000
Run 11	Bin 121	Error 41.04	Del 0.002
Run 12	Bin 121	Error 26.60	Del 0.004
Run 13	Bin 121	Error 17.85	Del 0.006
Run 14	Bin 121	Error 15.16	Del 0.008
Run 15	Bin 121	Error 18.79	Del 0.010
Run 16	Bin 121	Error 27.48	Del 0.012
Run 17	Bin 121	Error 42.57	Del 0.014
Run 18	Bin 121	Error 63.63	Del 0.016
Run 19	Bin 121	Error 90.78	Del 0.018

Shown below, are the 20 SPL curves re-created by the IHBT algorithm. Please note, that the vertical scale resolution has been set again to 2dB. The pink curve is the originally measured SPL, and it is our “reference” SPL curve, as before.

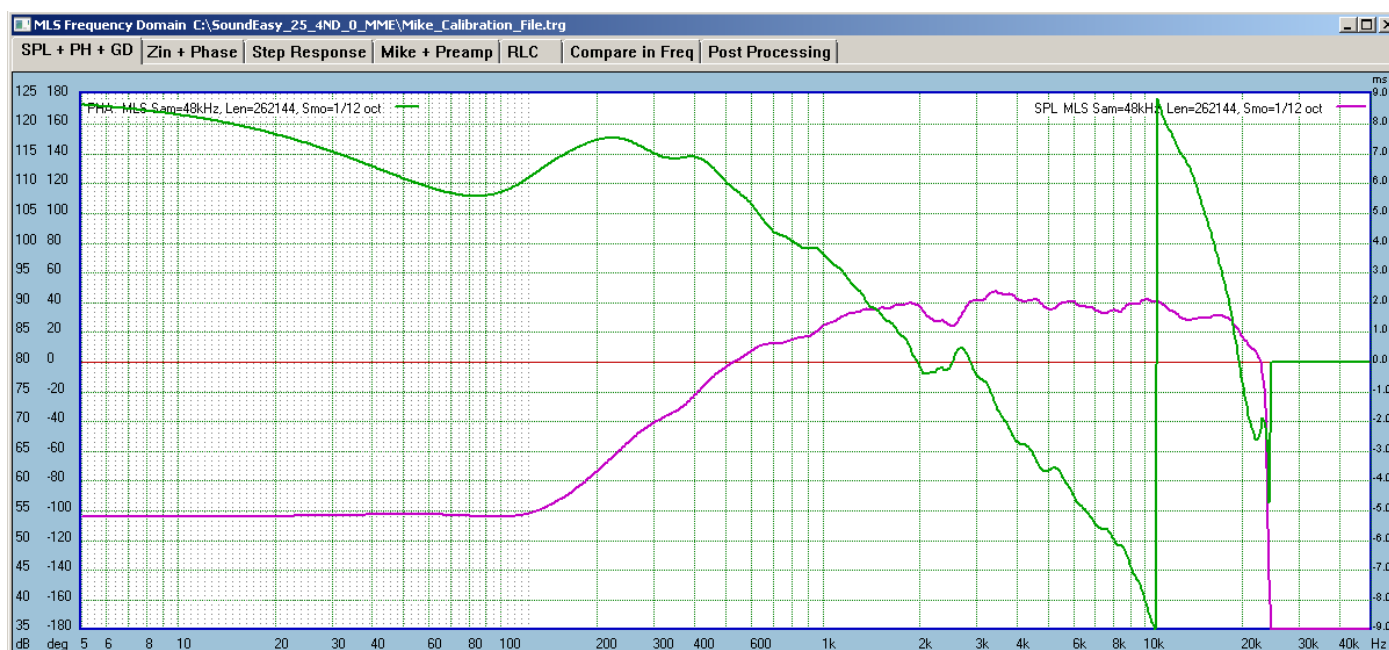
The process automatically searched for the minimum “Error” value and selected Run 14 (or time sample Bin 121 + Delay 0.008 ms) as the final parameters for placing the FFT window and calculating the resulting phase response



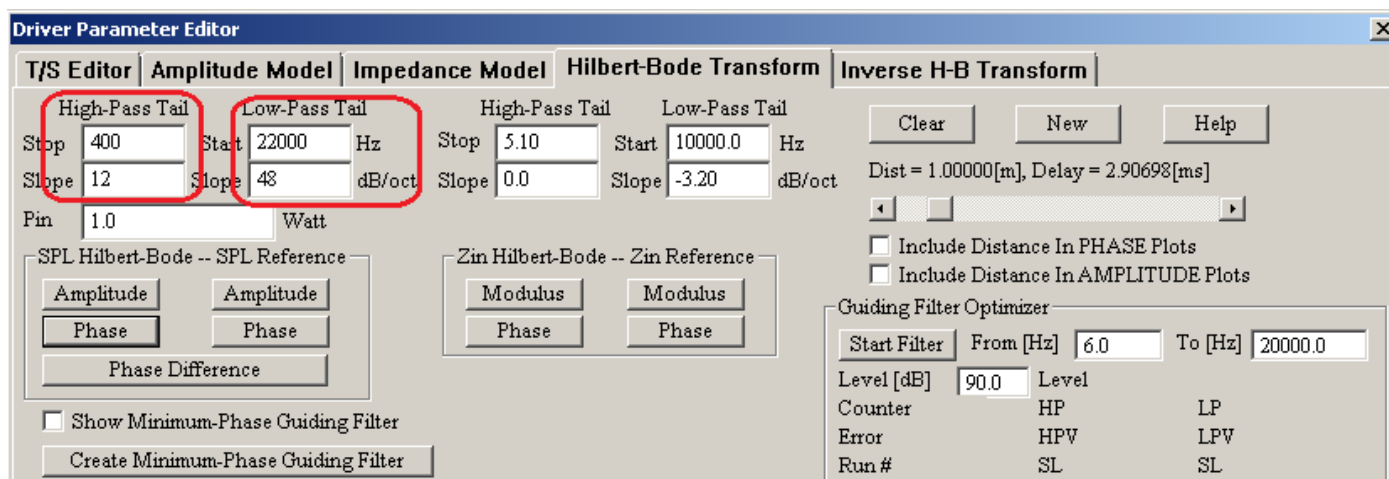
Back to MLS screen to correct FFT window placement at time sample Bin 121, with Pulse Delay of 0.008ms – see below.



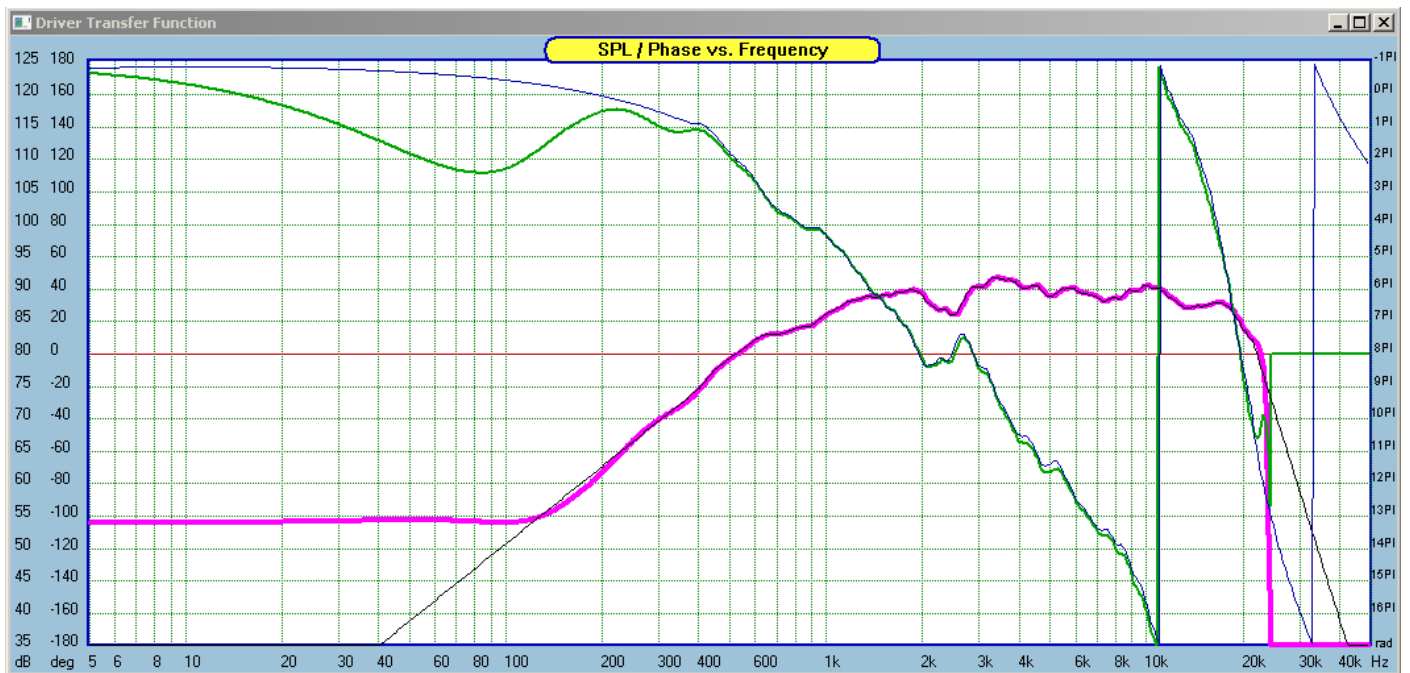
Now, we obtain **correctly extracted minimum-phase response** – see below.



Having done that, the measured phase response becomes the **PHASE TEMPLATE** for correcting HBT settings. We can now go to HBT screen and correct HBT settings to reflect correctly measured Minimum-Phase phase response.





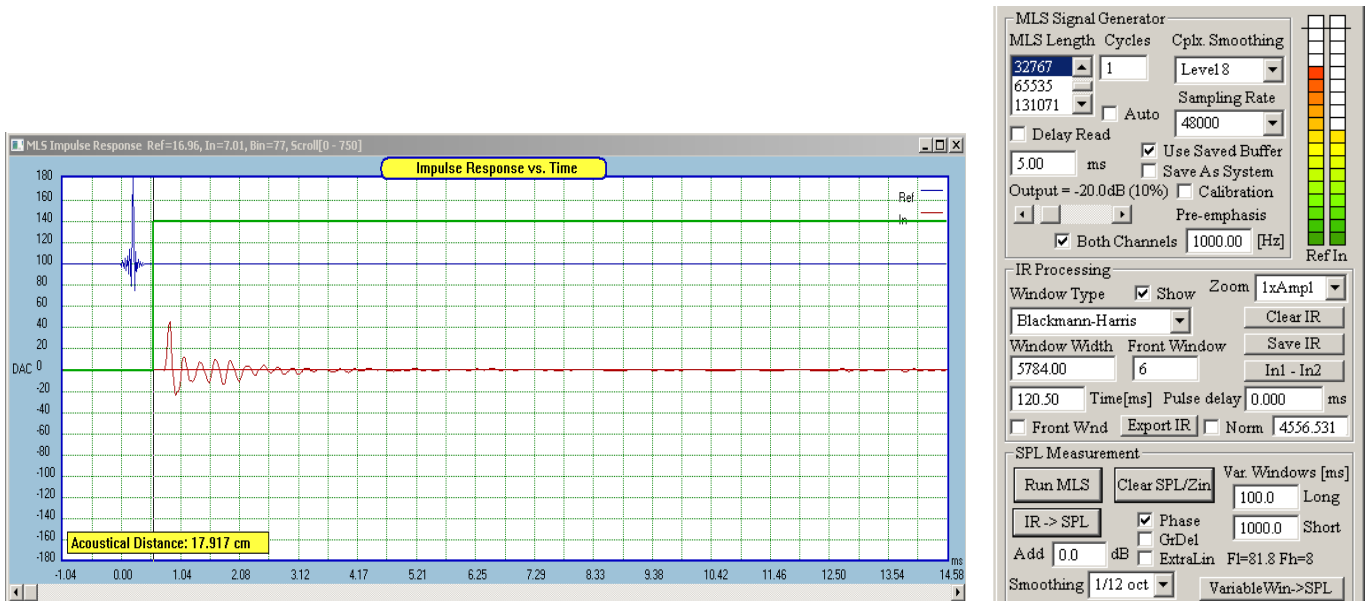


The thin blue curve is the minimum-phase phase response. Please note, that the phase response is clean and extends across the whole frequency range.

The thin black curve is the measured SPL response. Please note, that the SPL response is clean and extends across the whole frequency range.

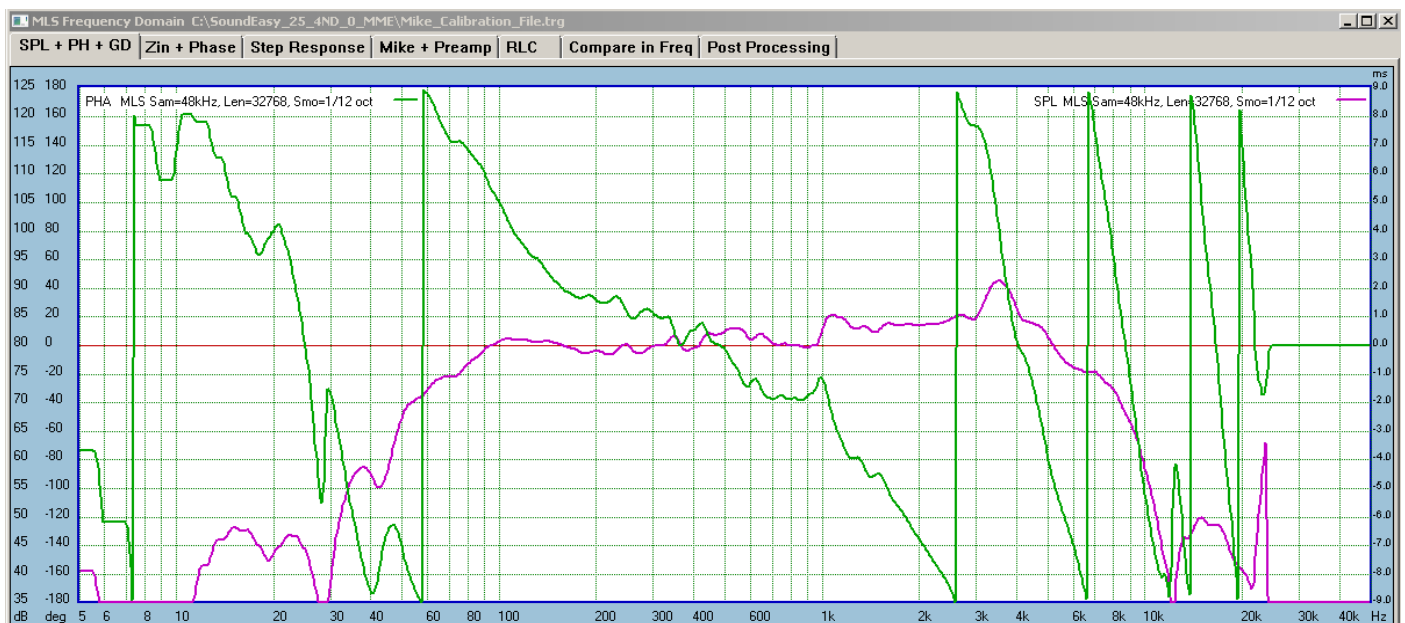
## 12" Woofer Driver in Vented Box

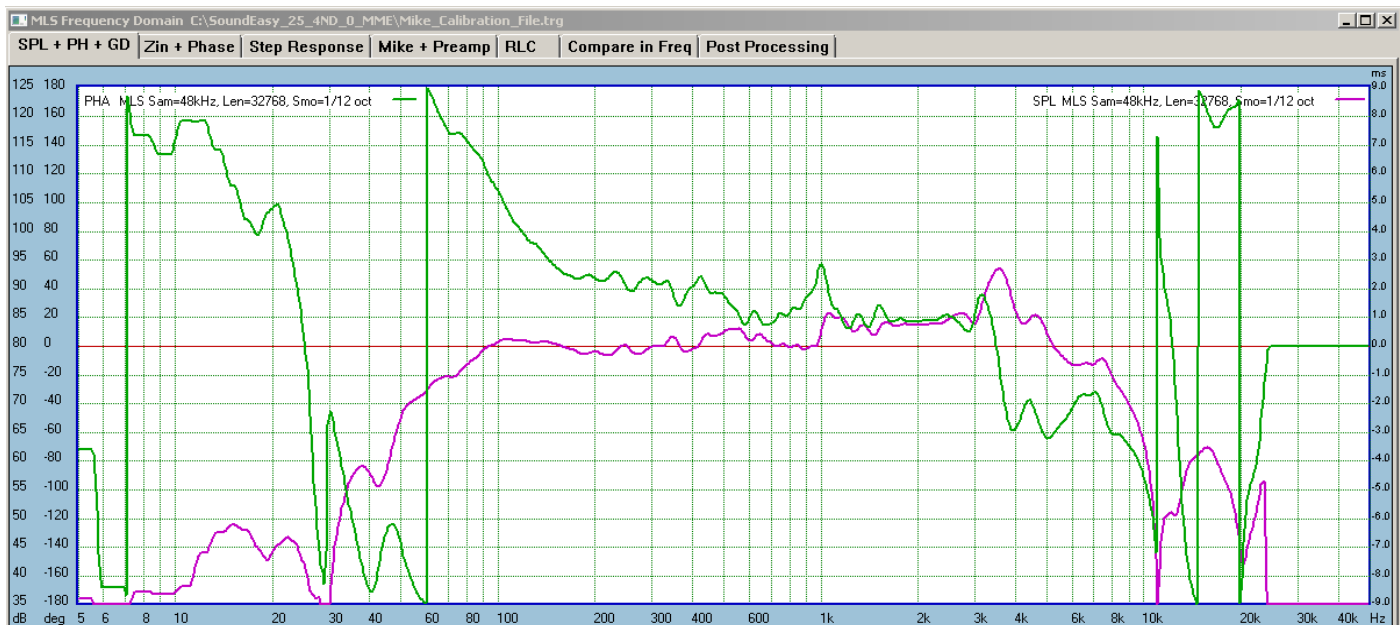
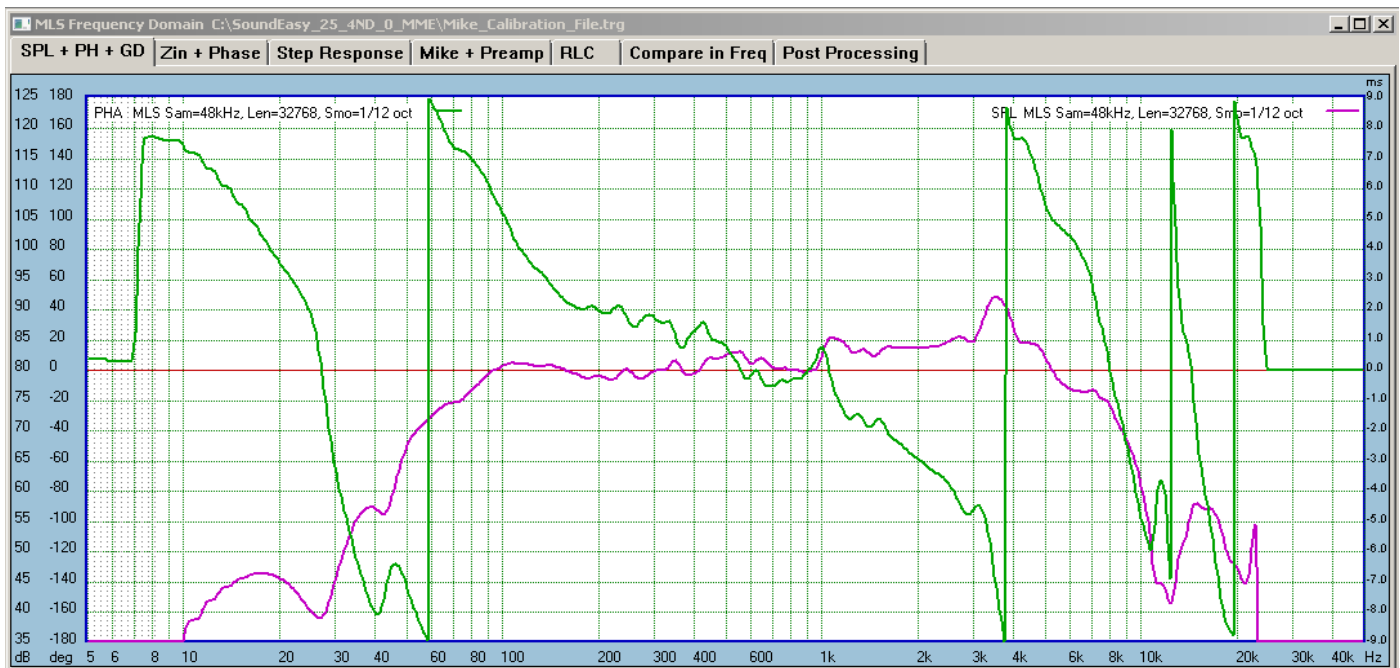
The next driver example is a 12" woofer driver measured in mid-sized room during daytime. Resulting SPL and Phase responses show room reflections and general low-level acoustic noise and reflections affecting measurement accuracy below -35dB.



Now, we are grappling with the same question as for the tweeter – **how to determine the correct FFT window placement?**. Shown below are three examples of FFT window placement, and corresponding phase response (green curves).

On the first picture, the FFT was placed at Bin 72, resulting in perhaps too many +180/-180 deg phase transitions. On the middle picture, the FFT window start was placed at Bin 77. Here, we have fewer phase transition – but is this correct?. On the bottom picture, the FFT window was placed at Bin 81, resulting in phase response with a very few transitions.





As explained in Part I article, the IHBT requires phase slopes to be attached to the phase response. There are several reasons for this.

1. At high frequency range, the measured phase response is limited to  $5\text{Hz} - \frac{1}{2}$  sampling frequency and does not extend to the screen limit.
2. At low frequency range, the SPL is heavily distorted by the effect of FFT windowing and reflections.
3. At both: high and low frequency ranges the measured SPL and phase responses accuracy is limited by the measurement system dynamic range. There are sound reflections, and background noises, that typically produce garbage SPL/Phase responses at the frequency fringes. This needs to be removed.

Therefore, attaching the asymptotic “phase tails” would clean up the measurements perfectly and would enable the SPL/Phase to be calculated over the whole frequency range of 5Hz-50kHz. As it was explained in Part I, the phase slopes are attached via SPL slopes mechanism – so, that the process looks more familiar to the user.

Most importantly, the SPL calculated via IHBT is not sensitive to the attached slopes, as it is calculated only between its designated frequency points, that are different from the attachment points of the phase slopes. In our example, the phase slopes are attached at 35Hz (+24dB equivalent) and 9.5kHz (-36dB equivalent). While the IHBT is calculated between 100Hz and 9.5kHz.

Phase from HBT using selected example slope parameters.

Driver Parameter Editor

T/S Editor | Amplitude Model | Impedance Model | Hilbert-Bode Transform | Inverse H-B Transform

High-Pass PHASE Tail Equivalent To SPL    Low-Pass PHASE Tail Equivalent To SPL

Stop 35 Hz    Start 9500.0 Hz

Slope 24 dB/oct    Slope 36 dB/oct

1. Phase Reference ----- 2. Inverse HBT

Amplitude From Measurement

Phase From Measurement    SPL From Selected Phase

Phase From HBT

Phase From Guiding Filter

☐ Show Minimum-Phase Guiding Filter

Create Minimum-Phase Guiding Filter

☐ Show Unwrapped Phase

☒ Show Measured Phase

☒ Show HBT Phase

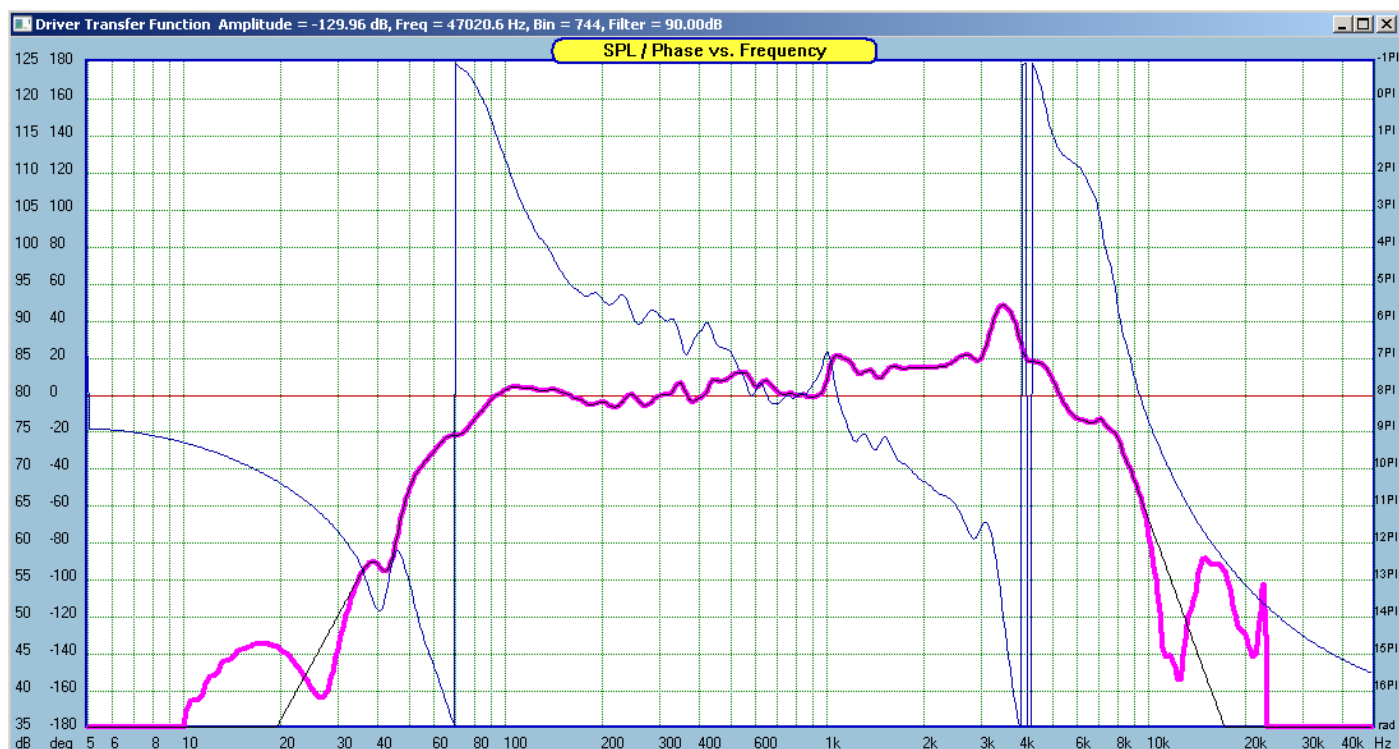
3. Inverse HBT Optimizer

Start Filter From [Hz] 35.0 To [Hz] 9500.0

Count = 41    Gain = 0.06    0.06

Error = 1.150    Angle = 0.68    0.68

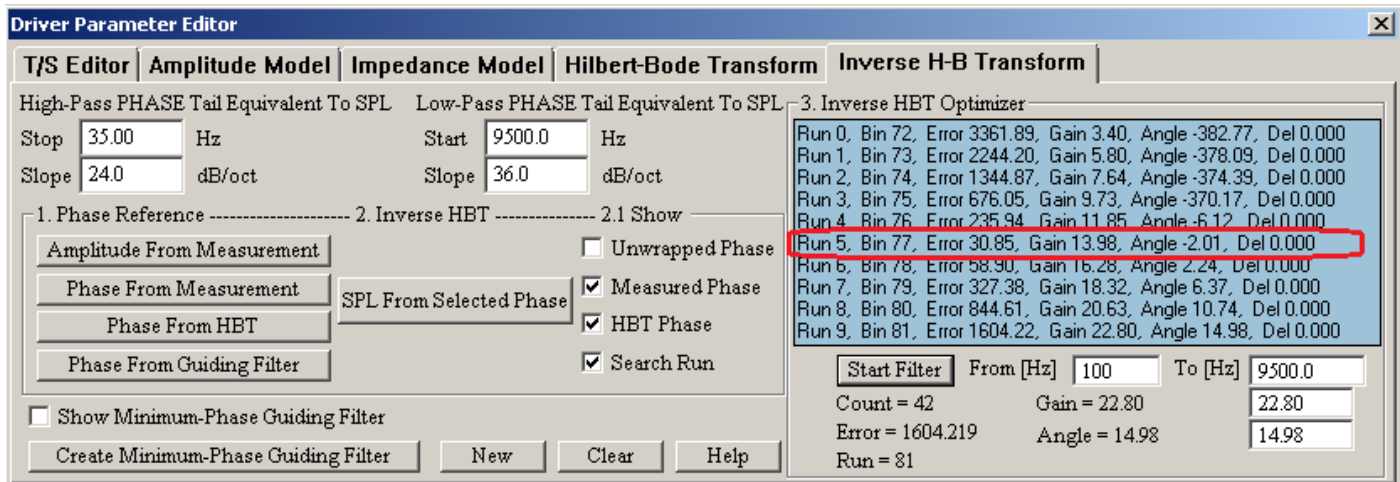
Run = 2





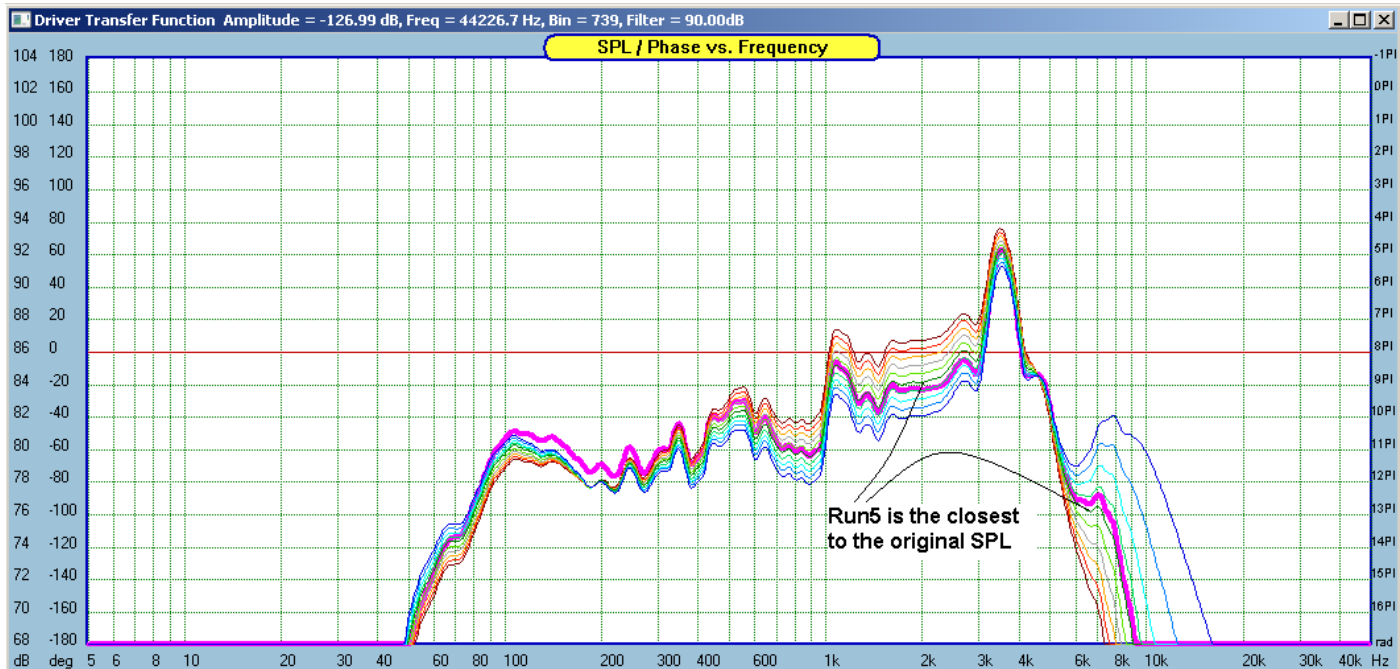
## Step 1 – 10 Coarse FFT window placements

As explained in the tweeter example before, in Stage I, the FFT window is placed at 10 arbitrary, but consecutive time samples, and resulting phase responses are supplied to the IHBT algorithm. This way, we obtain 10 SPL responses and one of them will be the best match with the originally measured SPL.



Shown below, are the 10 SPL curves re-created by the IHBT algorithm. Please note, that the vertical scale resolution has been enhanced from 5dB to 2dB in order to increase visual separation between SPL curves. The pink curve is the originally measured SPL, and it is our “reference” SPL curve.

Next, the process automatically searched for the minimum “Error” value and selected Run 5 (or time sample Bin 77) as the starting point for the Stage II.



Stage II starts automatically

## Step 2 – 20 Fine FFT (positive and negative) window delays

In Stage II, the algorithm adds or subtracts ten 2usec time delays so that the new phase is recalculated across 20usec to the left and 20usec to the right of the sampling time determined from Stage I.

Driver Parameter Editor

T/S Editor | Amplitude Model | Impedance Model | Hilbert-Bode Transform | Inverse H-B Transform

High-Pass PHASE Tail Equivalent To SPL    Low-Pass PHASE Tail Equivalent To SPL    3. Inverse HBT Optimizer

Stop 35.00 Hz    Start 9500.0 Hz

Slope 24.0 dB/oct    Slope 36.0 dB/oct

1. Phase Reference    2. Inverse HBT    2.1 Show

Amplitude From Measurement    ☐ Unwrapped Phase

Phase From Measurement    SPL From Selected Phase    ☒ Measured Phase

Phase From HBT    ☒ HBT Phase

Phase From Guiding Filter    ☒ Search Run

☐ Show Minimum-Phase Guiding Filter

Create Minimum-Phase Guiding Filter    New    Clear    Help

Start Filter    From [Hz] 100    To [Hz] 9500.0

Count = 23    Gain = 15.88    15.88

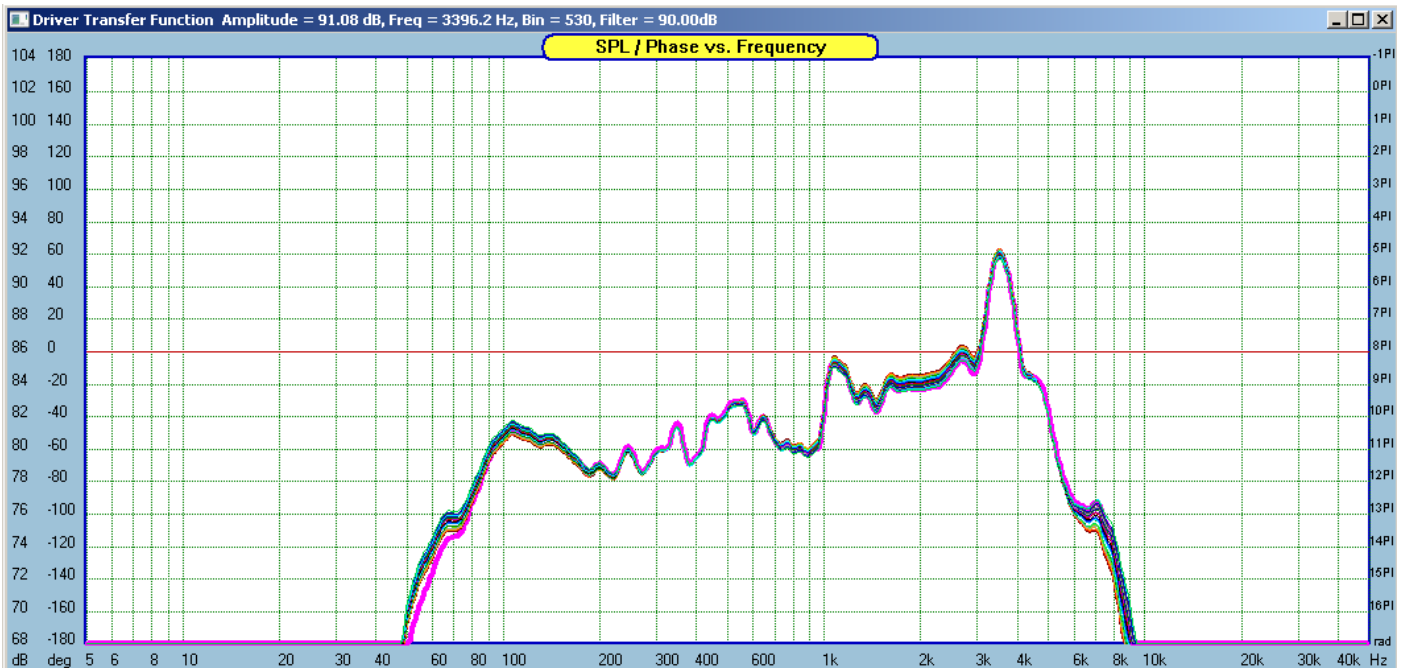
Error = 42.734    Angle = 1.76    1.76

Run = 77    Bin 77, Error 14.150, Del 0.008

Run	Bin	Error	Del
Run 0	Bin 77	Error 224.20	Del -0.020
Run 1	Bin 77	Error 195.14	Del -0.018
Run 2	Bin 77	Error 168.22	Del -0.016
Run 3	Bin 77	Error 143.46	Del -0.014
Run 4	Bin 77	Error 120.91	Del -0.012
Run 5	Bin 77	Error 100.49	Del -0.010
Run 6	Bin 77	Error 84.25	Del -0.008
Run 7	Bin 77	Error 66.28	Del -0.006
Run 8	Bin 77	Error 52.10	Del -0.004
Run 9	Bin 77	Error 40.34	Del -0.002
Run 10	Bin 77	Error 30.85	Del 0.000
Run 11	Bin 77	Error 23.38	Del 0.002
Run 12	Bin 77	Error 18.07	Del 0.004
Run 13	Bin 77	Error 15.16	Del 0.006
Run 14	Bin 77	Error 14.15	Del 0.008
Run 15	Bin 77	Error 15.49	Del 0.010
Run 16	Bin 77	Error 18.99	Del 0.012
Run 17	Bin 77	Error 24.73	Del 0.014
Run 18	Bin 77	Error 35.80	Del 0.016
Run 19	Bin 77	Error 42.73	Del 0.018

Shown below, are the 20 SPL curves re-created by the IHBT algorithm. Please note, that the vertical scale resolution has been set again to 2dB. The pink curve is the originally measured SPL, and it is our “reference” SPL curve, as before.

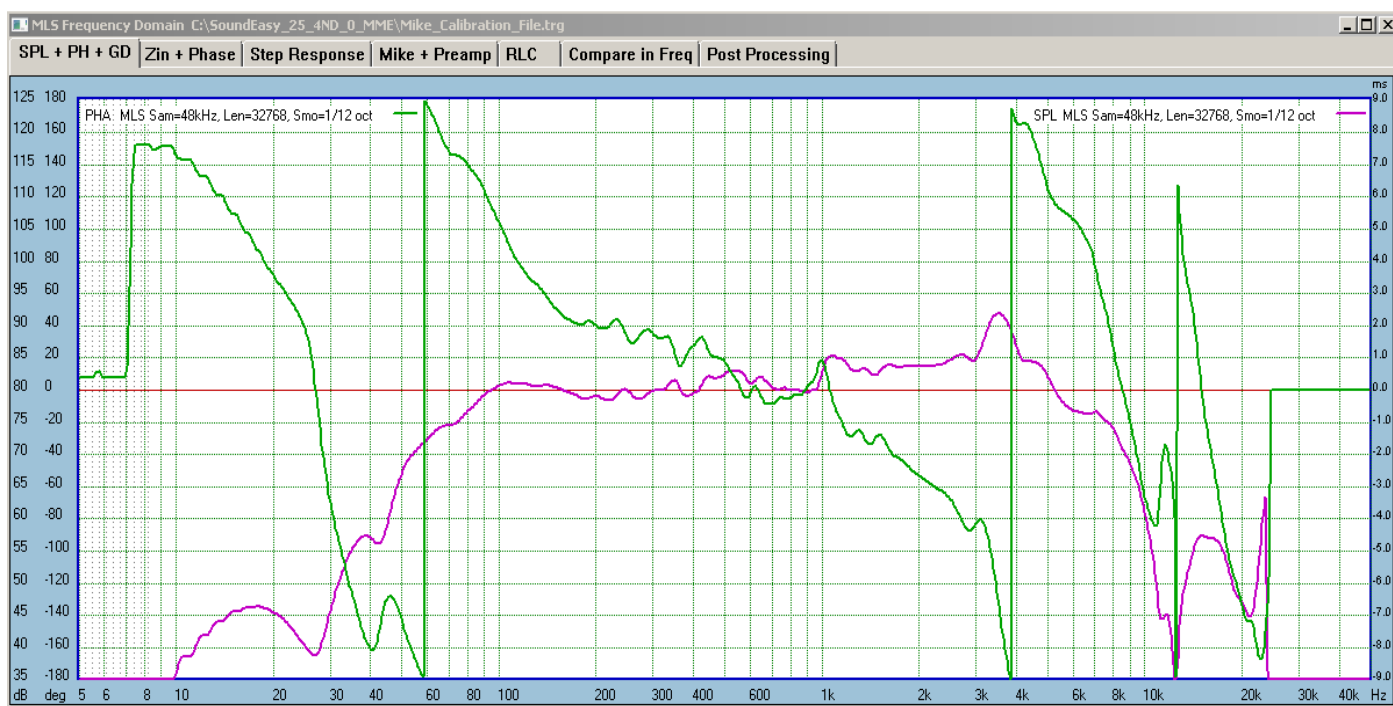
The process automatically searched for the minimum “Error” value and selected Run 14 (or time sample Bin 77 + Delay 0.008 ms) as the final parameters for placing the FFT window and calculating the resulting phase response



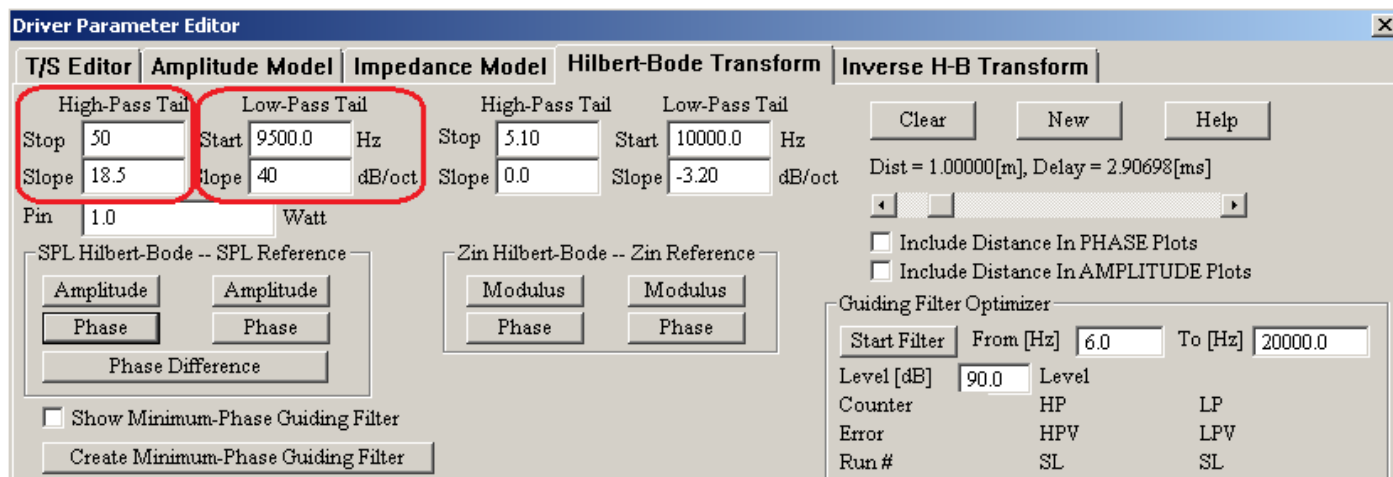
Back to MLS screen to correct FFT window placement at time sample Bin 77, with Pulse Delay of 0.008ms – see below.

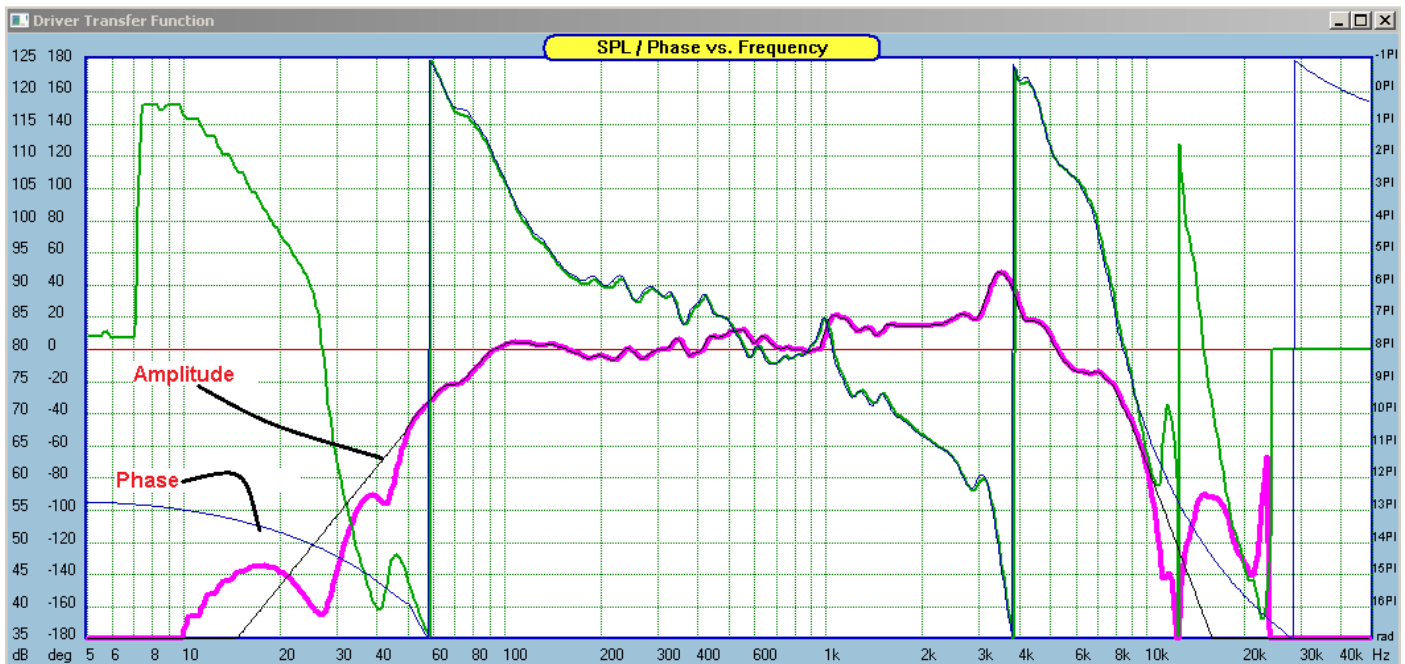


Now, we obtain **correctly extracted minimum-phase response** – see below.

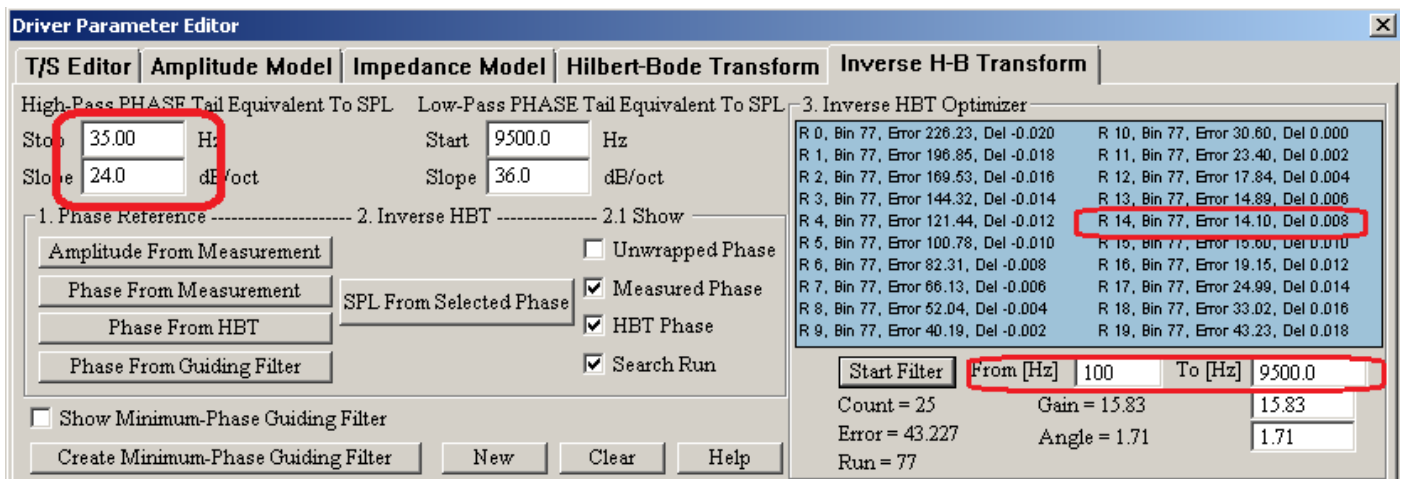
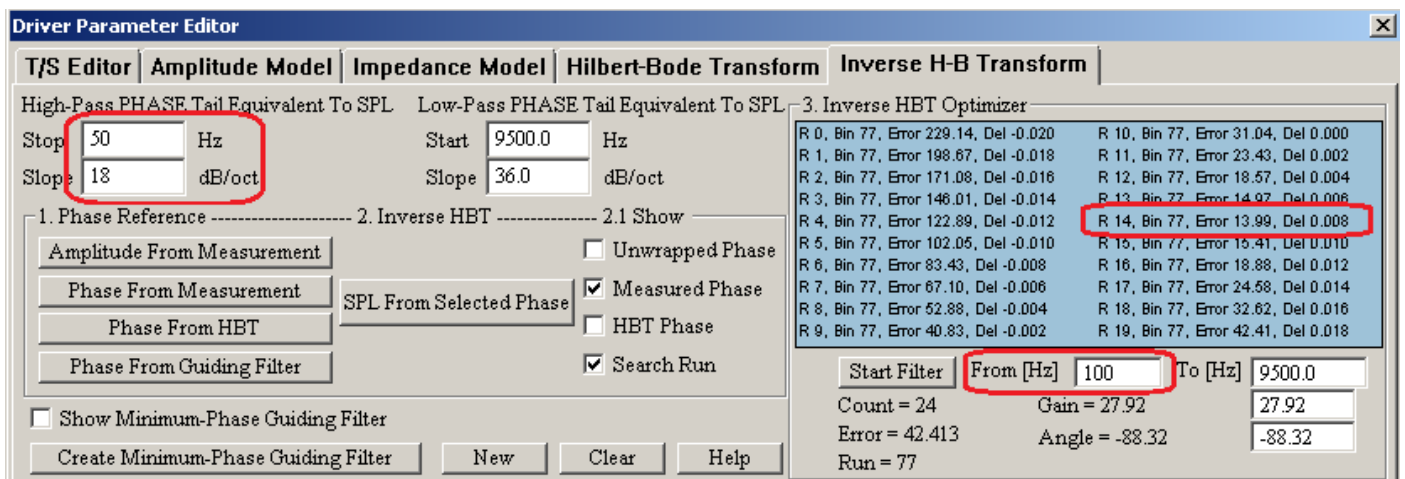


Having done that, the measured phase response becomes the **PHASE TEMPLATE** for correcting HBT settings. We can now go back to HBT screen and correct HBT settings to reflect correctly measured Minimum-Phase phase response.





As mentioned before, the IHBT is not sensitive to the attached slopes. We can see this on the three examples shown below. Despite having low-frequency phase slopes (actually SPL slopes translated to phase slope) attached at different frequency points and with different slopes, the algorithm always selects Bin 77 with the minimum error.





**Driver Parameter Editor**

T/S Editor | **Amplitude Model** | Impedance Model | Hilbert-Bode Transform | Inverse H-B Transform

High-Pass PHASE Tail Equivalent To SPL    Low-Pass PHASE Tail Equivalent To SPL    3. Inverse HBT Optimizer

Stop  Hz    Start  Hz  
Slope  dB/oct    Slope  dB/oct

1. Phase Reference    2. Inverse HBT    2.1 Show

Amplitude From Measurement    ☐ Unwrapped Phase  
Phase From Measurement    SPL From Selected Phase    ☒ Measured Phase  
Phase From HBT    ☐ HBT Phase  
Phase From Guiding Filter    ☒ Search Run

☐ Show Minimum-Phase Guiding Filter  
           

Start Filter    From [Hz]     To [Hz]   
Count = 18    Gain = 16.60    16.60  
Error = 29.933    Angle = 2.83    2.83  
Run = 77

R 0, Bin 77, Error 207.39, Del -0.020	R 10, Bin 77, Error 29.91, Del 0.000
R 1, Bin 77, Error 181.30, Del -0.018	R 11, Bin 77, Error 22.55, Del 0.002
R 2, Bin 77, Error 156.75, Del -0.016	R 12, Bin 77, Error 18.32, Del 0.004
R 3, Bin 77, Error 134.38, Del -0.014	R 13, Bin 77, Error 12.93, Del 0.006
R 4, Bin 77, Error 113.79, Del -0.012	<b>R 14, Bin 77, Error 11.06, Del 0.008</b>
R 5, Bin 77, Error 96.04, Del -0.010	R 15, Bin 77, Error 11.21, Del 0.010
R 6, Bin 77, Error 78.95, Del -0.008	R 16, Bin 77, Error 13.06, Del 0.012
R 7, Bin 77, Error 63.92, Del -0.006	R 17, Bin 77, Error 16.90, Del 0.014
R 8, Bin 77, Error 50.31, Del -0.004	R 18, Bin 77, Error 22.53, Del 0.016
R 9, Bin 77, Error 39.05, Del -0.002	R 19, Bin 77, Error 29.93, Del 0.018

## Conclusions

The two examples shown above demonstrate the following advantages of the IHBT Automated Method.

1. Loudspeaker drivers can be measured with accurately extracted minimum-phase phase response.
2. Final SPL and Phase responses are exceptionally clean and mathematically correct across the whole screen frequency range. They are locked together via HBT and IHBT transforms.
3. All undesirable background acoustical noises contaminating the measurements, are eliminated.
4. Having extracted minimum-phase phase response, it is easy to calculate corresponding **time-of-flight**. This would be  $\text{TOF} = \text{"number\_of\_bins"} / \text{"sampling frequency"} + \text{Added Delay}$ .
5. Knowing TOF and the speed of sound, we can calculate "Acoustical Distance"

**Acoustical Distance: 18.192 cm**

6. Now, if you measure physical distance between the microphone and the front mounting baffle, you can correctly calculate the "Acoustic Centre Offset" for each driver.

Thank you for reading

Bohdan