Chapter 3. Background

Parameters

The following parameters and formulas are used within BoxCad program.

General Parameters

pi = 3.142	pi number
ro = 1.18	density of air 1.18kg/m3
c = 344.8	speed of sound 344.8 m/s
eg = 2.83	generator output
u = 1.56/100000.0	kinematic coefficient of viscosity

Driver Parameters

Sw	piston area of the driver
aw	piston radius of the driver
Mmd	mass of the voice coil assembly
Cms	mechanical compliance
Rms	mechanical resistance of suspension
Mm1	mass of air on one side of diaphragm
Rm	Real part of air load
Xm	Imaginary part of air load
Ma1	radiation mass of the front of the driver
Rar1	resistance of the front side of the driver
Bl	BL factor of the driver
Fs	driver's free air resonance
Mad	acoustic mass of the voice coil assembly
Cas	acoustical compliance
Ras	acoustical resistance of suspension
Vas	effective compliance volume
Qe	electrical Q of the driver
Qm	mechanical Q of the driver
Qt	total Q of the driver
Re	electrical DC resistance
Rea	electrical DC resistance transformed to acoustical side
Le	VC inductance
Lea	VC inductance transformed to acoustical side
SPL	efficiency in dB's
pax	pressure at distance x

Enclosure Parameters

Cab	acoustic compliance of the box
Rab	real part of box impedance
Ral	enclosure leakage loss
Mab	air load mass at the back of the driver in the box
Vb	box volume

d	box depth
Sb	area of panel on which the driver is mounted
Fb	box tuning frequency
Qb	absorption loss of the box
Ql	leakage loss of the box
Qbt	total box loss

Port Parameters

t	length of port
ap	effective radius of port in m
Sp	effective area of port in m2
Map	acoustic mass of the air in port
Rap	acoustic resistance of the port
Ma2	acoustic mass of outer end of port
Rar2	resistance of the front side of port
Mat	Mat = Ma2 + Map
Qp	total Q of the port
Ma	Ma = Mad + Ma1 + Mab
Ra1	Ra1 = Rea + Ras + Rar1
Rat	Rat = Rap + Rar2

Formulas Used

```
Sw  = pi*aw*aw;
Qt  = Qm*Qe/(Qm+Qe);
Cms = Vas/(Sw*Sw*ro*c*c);
Rms = 1/(2*pi*Fs*Cms*Qm);
Mmd = 1/(2*pi*Fs*2*pi*Fs*Cms);
Mm1 = 2.67*aw*aw*aw*ro;

Mad = Mmd/(Sw*Sw);
Ras = Rms/(Sw*Sw);
Cas = Cms*(Sw*Sw);
Vas = Cas*(ro*c*c);
Fs  = 1/(2*pi*sqrt(Cas*Mad));
Rea = (Bl*Bl)/(Re*Sw*Sw);
Lea = (Le/(Bl*Bl))*Sw*Sw;

Ma1 = 0.27*ro/aw;
Rar1= (pi*Fs*Fs*ro)/c;
Ma  = Mad + Ma1;
Ra1 = Rea + Ras + 2*Rar1;
Qm  = (2*pi*Fs)*Mmd/Rms;          also  Qm = 1/(2*pi*Fs*Cas*Ras);
Qe  = 2*pi*Fs*Re*Ma*Sw*Sw/(Bl*Bl);
```

This Qe value is used by the program as consistency check on entered driver's parameters. If force factor (BL) or piston radius (ap) are wrong, the Qe calculated by the program will be much different from Qe entered.

```
Qt  = Qm*Qe/(Qm+Qe);                also Qt  = (2*pi*Fs)*Ma/Ra1;
pax = 2*(eg*Bl*ro)/(Re*Ma*4*pi*Sw*1.0);  ( in half-space )
Rm  = (Bl*Bl)/Re + Rms + 2*(2*pi*Fs*2*pi*Fs*Sw*Sw*ro)/(2*pi*c);
Xm  = 2*pi*Fs*(Mmd + 2*Mm1) - 1/(2*pi*Fs*Cms);
pax = (eg*Bl*Sw*Fs*ro)/(1.0*Re*sqrt(Rm*Rm +Xm*Xm));
SPL = 20*log10(pax/0.00002);      ( referenced to Po )

Sp  = pi*ap*ap;
Ma2 = (0.6*ap*ro)/Sp;
Map = (t + 0.6*ap)*ro/(pi*ap*ap);  ( includes inside loading )
Mat = Ma2 + Map;

Mab = (ro*d*Sw)/(3*Sb*Sb) + 8*ro*(1 - Sw/Sb)/(3*pi*sqrt(pi*Sw));
Cab = Vb/(ro*c*c);
Fb  = 1/(2*pi*sqrt(Cab*(Mat)));
Qb  = 1/(2*pi*Fb*Cab*Rab);
Rab = 1/(2*pi*Fb*Cab*Qb);
Rap = (ro/Sp)*sqrt(2*2*pi*Fb*u)*(t/ap + 1);
Rar2= (pi*Fb*Fb*ro)/c;
Rat = Rap + Rar2;
Qp  = (2*pi*Fb)*Mat/Rat;
Ql  = 2*pi*Fb*Cab*Ral;
Qt  = 1/(1/Qp+1/Qb+1/Ql);    also   Qt  = 1/(1/Qp+1/Qb);
```

Electrical Impedance

```
Cmes =  Mad*(Sw*Sw)/(Bl*Bl);       Lces  =  Cas*(Bl*Bl)/(Sw*Sw);
Res =  (Bl*Bl)/(Sw*Sw*Ras);        Cmer =  Ma1*(Sw*Sw)/(Bl*Bl);
Rer  = (Bl*Bl)/(Sw*Sw*Rar1);       Lecb = Cab*(Bl*Bl)/(Sw*Sw);
Reb  =  (Bl*Bl)/(Sw*Sw*Rab);       Rel  =  (Bl*Bl)/(Sw*Sw*Ral);
Cmeb =  Mab*(Sw*Sw)/(Bl*Bl);

Cmep =  Map*(Sw*Sw)/(Bl*Bl);       Cmep2= Ma2*(Sw*Sw)/(Bl*Bl);
Rep  = (Bl*Bl)/(Sw*Sw*Rap);        Rer2 = (Bl*Bl)/(Sw*Sw*Rar2);
```

Dot Method

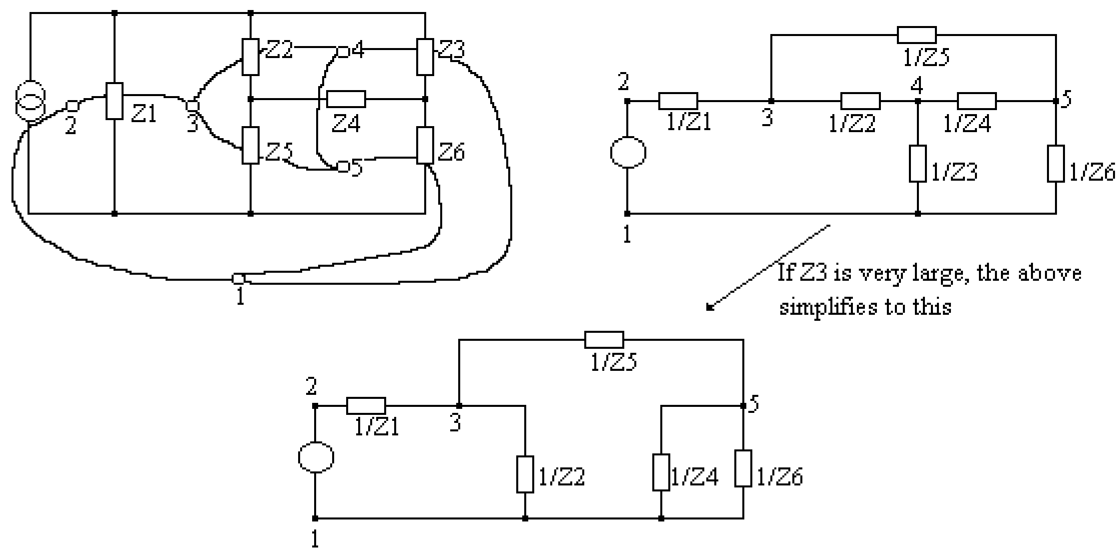
Dot Method is a unique tool for transforming one circuit representation into another. In all cases presented in this chapter, enclosure model was based on mechanical mobility representation. This is possibly the easiest starting point because components of the model are direct representation of physical properties of the enclosure. Since acoustical impedance representation is more “manageable” via electronics CAD systems, a conversion needs to take place. The mechanical mobility and acoustic impedance circuits are “dual” to each other. The Dot Method works as follows:

1. Place a “dot” in each loop and assign consecutive number to each dot - see circuit on the left below.

2. Connect each “dot” with a link passing through the component between dots. In the circuit on the left, the outside dot was assigned number 1 and all internal loops were assigned remaining consecutive numbers.

3. Now, the dual circuit can be created. The dots in the original circuit become nodes in the dual circuit representation and all components are inverted. On the circuit on the right the external dot becomes ground node, dot 2 becomes node 2 and so on. Components crossed by the links are connected between their assigned nodes so that: component on the link between dot 2&3 is now inverted and connected between node 2&3, component on the link between dot 3&4 is now inverted connected between node 3&4. When this process is completed, the circuit should look like the one on the right hand side.

It may be convenient to simplify the dual circuit at the same time. For example, when a very large impedance (eg: Z3) is inverted (1/Z3), it becomes almost a short circuit connecting node 4 to node 1, resulting in the circuit shown in the middle.



Transfer functions

Sound pressure P at a distance r is proportional to frequency f and is governed by the following formula:

$$P_r = \frac{f * \rho_0}{2 * r} |U|$$

Quantity U is volume velocity, r is the distance in meters, f denotes frequency and ρo=1.18kg/m3 is density of air. Cone excursion X is described by the following formula:

$$X = \frac{1}{s * Cas * Zat}, \ s = j\omega$$

Zat is the driving point impedance of the acoustical impedance representation and ω=2πf is the angular frequency. Cas is the acoustic compliance of the driver.

Two plotting windows can be invoked from the main menu: (1) frequency domain analyser (“Frequency” option) and (2) time domain analyser (“Time” option). It is mandatory to select appropriate plotting option for given type of schematic (model) being analysed.

Scaling of the curves, component values and more importantly, formulas for which the input is provided by the standard CAD functions, demand attention to the correct selection of the transfer function types.

In the **frequency domain window**, the following plots are available from the floating menu invoked by the "Plot" button:

1. PASSIVE DRIVER MODEL

1.1 Amplitude - Plots amplitude response. Works only with acoustic impedance model and requires the user to enter branch numbers (blue numbers on the schematic). As explained before, currents in this model represent volume velocity. Before the curve is plotted, a dialogue box allowing for the branch numbers to be entered, will be presented to the user. Acoustic impedance model is characterized by large inductors, 1-100kohm resistors and 1-1000uF capacitors. Component values can be conveniently obtained using provided calculators. Amplitude response is saved automatically for use in combination with active circuit frequency response. Amplitude response is plotted as an absolute SPL level. That means, if 1 Watt of power is fed into the driver with 92dB SPL, the curve will be plotted around 92dB level. If 10 Watts is fed in to the driver, the curve will be plotted at 102dB level, and so on. The input power level is selected from the "Power Compression Calculator".

1.2 Phase - Plots phase response. Other as in 1.1.

1.3 Impedance - Plots driving point input impedance of the network. Works with standard, electrical impedance models.

1.4 Imped. Phase - Phase response of the network's input impedance. Other as in 1.3.

1.5 Group Delay - Group delay of the acoustic impedance network. Other as in 1.1.

1.6 Cone Excursion - Cone excursion of the acoustic impedance model. Other as in 1.1.

2. STANDARD NETWORK

2.1 Amplitude - Plots amplitude response of a standard active/passive electrical network and requires the user to enter **node** numbers (red numbers on the schematic).
2.2 Phase - Plots phase response of a standard active/passive electrical network.

3. COMBINED

3.1 Amplitude - Plots amplitude response of an acoustic impedance network plotted and saved previously and standard network combined. Assumes that acoustic impedance model has been used prior and its amplitude response was saved (this is done automatically, when the curve is plotted) - see item 1.1. Typical usage includes powered subwoofer design, where the user would model the driver+enclosure using plots 1.1 to 1.6 and then design active equalization network.

3.2 Phase - Plots phase response of a combined network. Other as in 3.1.

3.3 Excursion - Plots cone excursion of a combined network - passive network and an amplifier connected to it. First, you must invoke or create acoustic impedance model and plot cone excursion at the required power level - plot 1.6. The curve will be saved automatically. Next, design the active compensation network to be used, and you are ready to plot “combined” cone excursion. This is one of the most important plots as it indicates potential mechanical damage to the “over equalised” driver.

4. Clear - Clears plotting and legend fields.

In the **time domain window**, the following plots are available from the floating menu invoked by the “Plot” button:

1. FAST FOURIER TRANSFORM

1.1 Acoustic Impedance FFT - Works only with acoustic impedance model and requires the user to enter **branch** numbers (blue numbers on the schematic).

1.2 Standard Network FFT - Works with standard active/passive network model and requires the user to enter **node** numbers (red numbers on the schematic).

1.3 Combined System FFT - Plots time response of an acoustic impedance network plotted and saved previously using **Acoustic Impedance FFT** option and standard network combined. Assumes that acoustic impedance model has been used prior and its amplitude response was saved. Typical usage includes powered subwoofer design, where the user would model the driver+enclosure using frequency domain plots 1.1 to 1.6 and then design active equalization network.

2. MODIFIED NODAL METHOD

2.1 Acoustic Impedance Network - Works only with acoustic impedance model and requires the user to enter branch numbers (blue numbers on the schematic).

2.2 Standard Network - Works with standard active/passive network model and requires the user to enter branch numbers (blue numbers on the schematic).

3. Test Pulse - Plots bi-polar test pulse.

4. Clear - Clears plotting and legend fields.

Period of the test pulse can be indirectly changed from the “Test Frequency” filed provided in the right-bottom corner of the time domain window. One full period of the system’s response is always plotted on the screen.

Note: Prior plotting frequency and time response curves, a pop-up dialogue box offers the user an opportunity to change vertical scaling on the plotting window. Built-in default values are recommended starting point. Group Delay and Cone Excursion scales are the only one, that accept real numbers for scale resolution. All other scales must be entered as integer numbers.

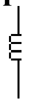
Mechanical Mobility

In this type of analogy, velocity ‘u’ corresponds to voltage and pressure ‘p’ corresponds to current.

1. Mass Mm (kg).

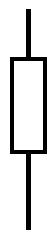
Second terminal of mass is the earth and has always zero velocity.

2. Compliance Cm (m/N).



Compliant elements usually have two apparent terminals, which move with different velocities.

3. Resistance Rm (mks mechanical ohm).



Resistive elements usually have two apparent terminals, which move with different velocities.

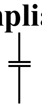
Acoustical Impedance

In this type of analogy, pressure ‘p’ (newtons per square meter) corresponds to voltage and volume velocity ‘U’ (cubic meters per second) corresponds to current.

1. Mass Ma (kg/m).

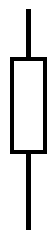
Mass of air accelerated by a force, but not compressed. Typical example is a tube with cross-sectional area S, filled with gas (vent). $M_A = M_M / S^2$

2. Compliance Ca (m/N).



Compliance is associated with a volume of air compressed by a force, but not accelerated. $C_A = C_M * S^2$. One terminal must always be at ‘ground potential’.

3. Resistance Ra (mks acoustic ohm).



Resistive elements usually have two apparent terminals. Elements are associated with dissipative losses of gas movement. $R_A = R_M / S^2$

Representation

Mechanical mobility representation was chosen as a starting point, because it may be easier to identify individual components. The following observations are useful:

1. all the symbols have two terminals, having velocities Va and Vb.
2. symbol of the mass has one terminal connected to zero velocity, Vb=0.
3. terminal vibrate in vertical direction.
4. horizontal, massless, rigid and lossless line connects terminals with the same velocities.

When faced with the problem of drawing an equivalent electrical circuit for a mechanical system, it is helpful to determine whether the velocity across different elements is the same or not. This could help to determine if the elements should be in series or parallel. In the loudspeaker's mechanical mobility circuit, velocity is represented by potential, so mass, compliance and resistance of the vibrating assembly must be in parallel, as they move with the same velocity. Also, it is beneficial to consider both pressure and volume velocity for each element. For example, enclosure and vent are subjected to the same pressure, so in the mobility circuit they will appear in series, as the pressure (current) flaws through them. Once the mechanical mobility (sometimes called "inverse") representation is completed, the diagram is transformed to acoustical impedance, or "direct" circuit, using duality or "dot" method.

Direction of Current

In order to obtain all required circuit transfer functions, the module employs Modified Nodal Method. One of the components used by the method is Incidence Matrix A[][], describing direction of currents in the circuit. The following rules are adopted for matrix A[][]:

1. Current always flaws into the Input node.
2. For all other nodes, current always flaws from lower to higher node number.

In the acoustical impedance convention, volume velocity is represented by current, so the above will help to determine + and - signs in front of the currents when building the transfer function formula in the dialogue box.

Plotting Control

Frequency Domain and Time Domain plotting screens have been equipped with a plotting control dialogue boxes - separate for dealing with volume velocities (branch currents) and nodal voltages. The box pops-up after the you select desired curve from the floating menu. It has been recognized, that more complex designs may be coupled to the surrounding air via more than one volume velocity path (eg: vented box uses two paths). Therefore up to four paths (Bx data fields) can be specified from the dialogue box and can be arranged in a suitable formula.

For example: first Bx field is equal to 3
second Bx field is equal to -7
third Bx field is equal to 9
fourth Bx field is equal to -10

Hence: **Volume Velocity = branch 3 - branch 7 + branch 9 - branch 10**

When two drivers connected in parallel are modeled, they augment each other's acoustic output via mutual radiation impedance mechanism, adding up to 3dB of efficiency at lower end of the spectrum. BoxCad will modify system response accordingly to the dialogue box settings. The "Frequency Domain" screen dialogue box provides two check boxes for selecting single driver or 2 drivers in parallel configuration and a data field for entering separation between the acoustic centers of the two drivers. The middle section of the box contains data fields for scaling the plots. The "Cas" data field comes handy when plotting cone excursion and the "Pgen" is the calculated strength pressure generator applied to the input of the acoustic impedance model. The "Time Domain" screen dialogue box is similar, but the scaling control fields are replaced by one field - ratio of "Vout/Vin". Number of pulses to be plotted is controlled by the number entered in the first field of the "Scale Factors" box. Additionally, when you use Modified Nodal Method, you can choose to have a sequence of pulses plotted or a single pulse from 1 to 10. See Fig 3.1.

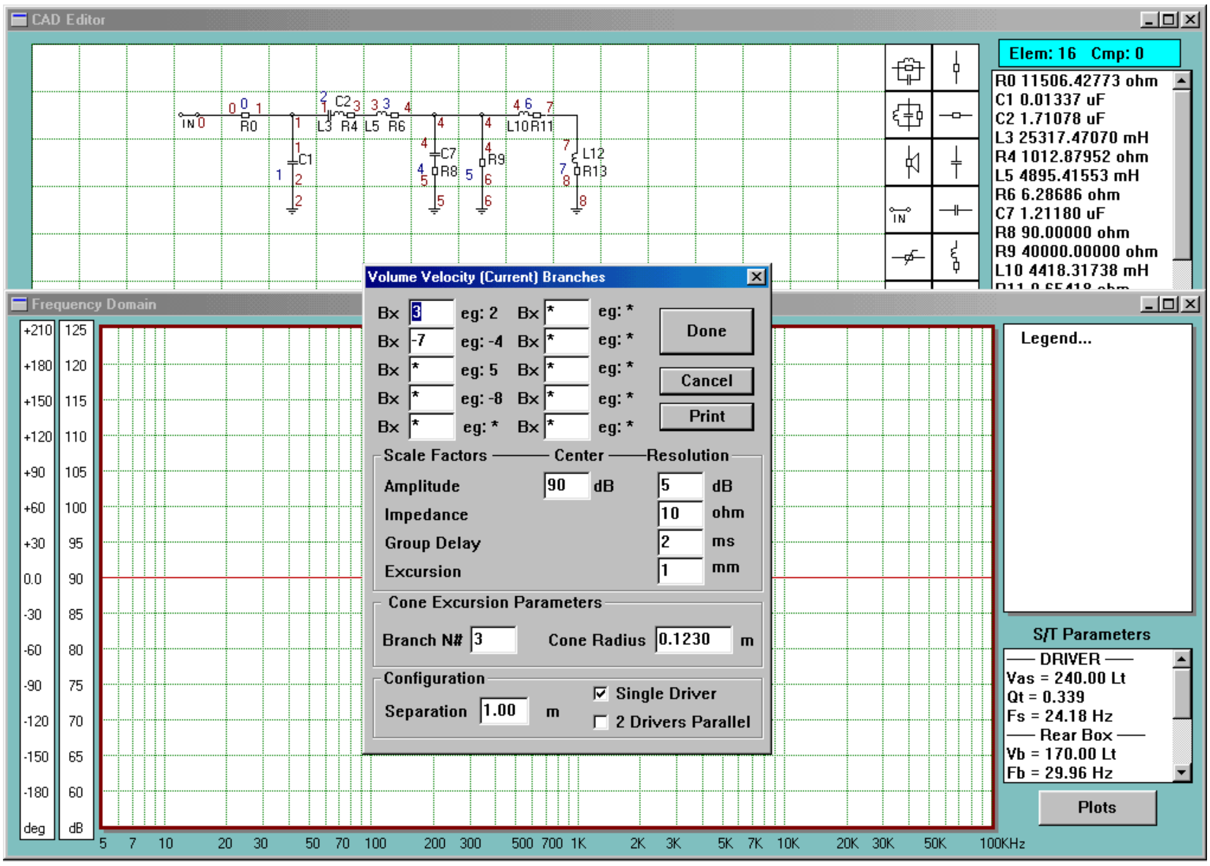


Fig 3.1 Frequency domain plotting control box

Cone Excursion Plots

Cone excursion, Xm, plots are performed accordingly to the following process:

1. Branch containing the cone element is selected. In Fig 3.2, Branch N# 3 is selected, because it contains all three elements representing driver's mass, loss and suspension. This way, volume velocity of the cone, **Uc**, (current in our schematic) in this branch is calculated.
2. Next, the program will divide **Uc** by the cone area, Sc, to obtain velocity, **uc**, of the cone.
3. Finally, the resulting velocity value is integrated (or divided by **s=jω** in frequency domain), to obtain cone excursion – see plot 1 on Fig 4.10.

Cone Excursion Xm = Uc/(s*Sc)

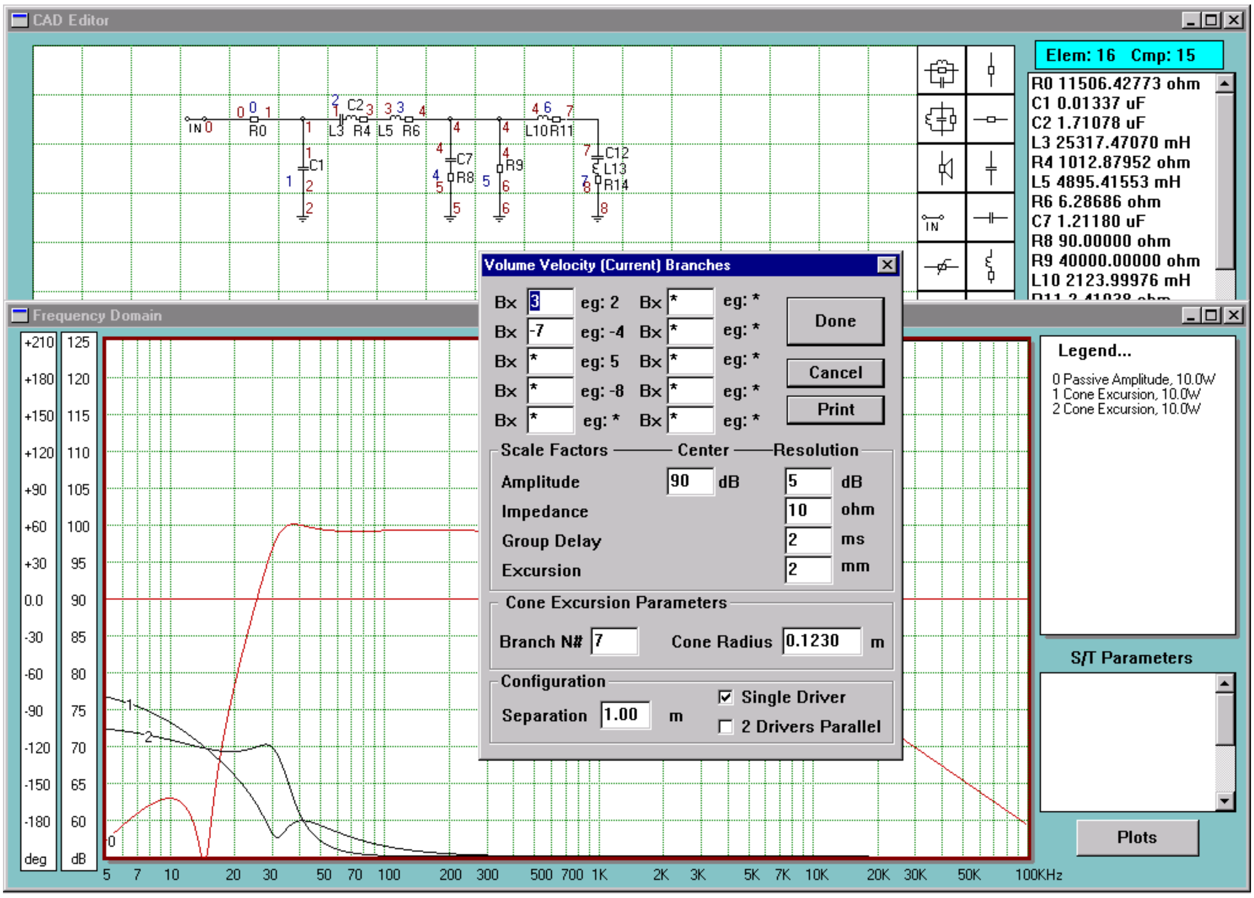


Fig 3.2 Control box for amplitude/cone excursion plots

The above process is quite universal, allowing you to plot passive radiator cone excursion as well. All you need to do, is to enter the branch number containing passive radiator element and then enter PR cone radius. Fig 3.2 shows Branch N# 7 was selected for the PR and the radius was the same as for the driver – see plot 2.

Generally, please select the following models:

1. SPL amplitude plots	**select**	**Acoustic Impedance Models**
2. SPL phase plots	**select**	**Acoustic Impedance Models**
3. SPL group delay plots	**select**	**Acoustic Impedance Models**
4. Impedance amplitude plots	**select**	**Electrical Impedance Models**
5. Impedance phase plots	**select**	**Electrical Impedance Models**
6. Cone excursion plots	**select**	**Acoustic Impedance Models**

Combined Plots

Combined plots are useful for modeling loudspeaker with electronic active equalization (EQ) circuits, such as active subwoofers of any kind. Of primary interest here would be the total SPL and cone excursions for various input power levels.

Our starting point here is the acoustic impedance circuit. You can use one of the built-in systems or generate your own schematic and component values. When finished, please save the schematic in datafile using “Models” -> “Save as Acoustic Impedance” menu option. This model will be suitable for modeling SPL and cone excursions.

Next step is to plot the actual parameter you wish to review. Please open the "Frequency Domain" plotting window and plot amplitude response and cone excursion curves. Button "Plots" on the "Frequency Domain" screen invokes a floating menu allowing you to select amplitude, phase or cone excursion plots of the acoustical impedance circuit. Plotting control dialogue box calls for volume velocity branch(s) (or current branch on our schematic) to be nominated for plotting of the amplitude and cone/PR excursion.

Input power level (Pin) can be changed from "Calculators" -> "Thermal Analysis" menu option, prior plotting the curves. Please remember, that increasing Pin will cause several of the drivers' T/S parameters to be changed. All these parameters need to be re-entered in the calculators to obtain updated corresponding circuit component values. Plotting curves causes the calculated responses to be saved automatically in memory for further use with the EQ circuit.

Now, we need to design the active EQ circuit. Please clear the CAD screen using "Edit" -> "Clear CAD" menu option and enter your circuit into the CAD screen. Once again, when you finish drawing the schematic and allocating component values, please save the circuit in datafile using "Models" -> "Save as Standard Network" menu option. Having created both, acoustic impedance circuit and electrical EQ circuit, we recommend, that you save them to disk for future reference. You are now ready to plot the Combined Plots. Button "Plots" on the "Frequency Domain" screen invokes a floating menu allowing you to select amplitude, phase or cone excursion plots of the combined (driver + EQ) circuits.

Cone/Vent/PR Velocity Plots

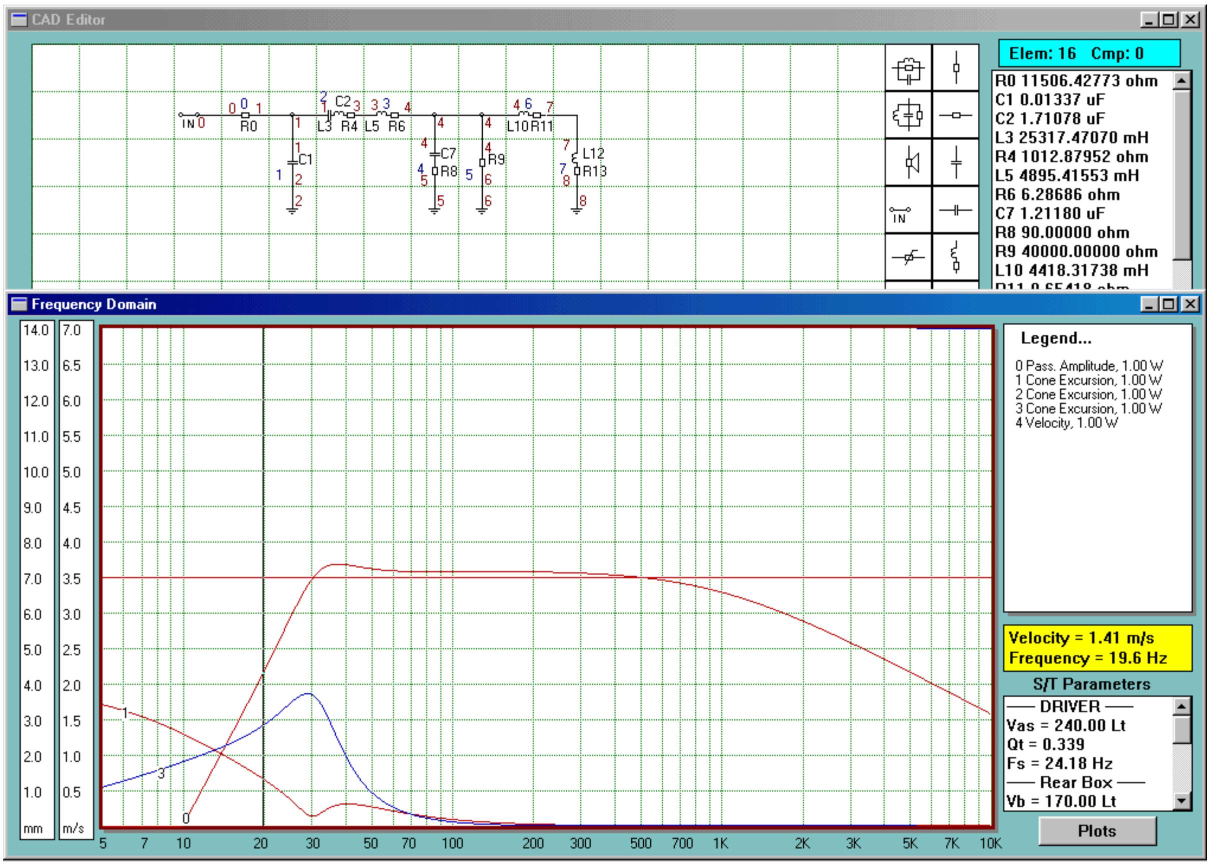


Fig 3.3 11cm vent is well into turbulent flow with the air velocity above 0.314m/s

Air flow occurs only when there is a difference between pressures. Air will flow from a region of high pressure to one of low pressure - the bigger the difference, the faster the flow.

When flow is low velocity and through narrow tubes, it tends to be more orderly and streamlined and to flow in a straight line. This type of flow is called laminar flow. Laminar flow is directly proportional to the driving pressure, such that to double the flow rate, one need only double the driving pressure. Laminar flow can be described by Poiseuille's Law.

When air flows at higher velocities, especially through a tube with irregular walls, flow is generally disorganized, even chaotic, and tends to form eddies. This is called turbulent flow. A relatively large driving pressure is required to sustain turbulent flow. Driving pressure during turbulent flow is in fact proportional to the square of the flow rate such that to double the flow rate one must quadruple the driving pressure. This turbulence increases the resistance dramatically so that large increases in pressure will be required to further increase the volume flowrate.

REYNOLDS NUMBER (Re)

Definition :
Reynolds Number is the ratio of the inertial forces to viscous forces acting on a fluid. It provides information about the flow behavior of the fluid, i.e. whether the flow is laminar or transitional. It is defined:

$Re = (D*V\rho)/\mu$

Where: D = diameter of the tube
V = average velocity of the air
ρ = density of air = 1.18kg/m^3
μ = viscosity of air = 1.8*10^(-5)

In general:
Re < 2000 **Laminar Flow**
2000 < Re < 4000 **Transitional Region**
Re > 4000 **Turbulent Flow**

The resistive force in the vent is proportional to velocity for laminar air flow, but proportional to the square of velocity for turbulent flow. **Therefore turbulence in the vent creates distortion.** For instance, the flow in a 4.3-inch diameter vent becomes turbulent for a peak excursion of more than 2.5 millimeters or so, at 20 Hz. So the vent flow is well into the turbulent region for any higher power level:

Example:
Xm = 2.5mm = 0.0025m
f = 20Hz
D = 0.11m or 4.3”
V = jωf Xm= 2*3.142*20*0.0025 = 0.314m/s
Re = [(0.11*0.314*1.18)/1.8]*10^5 = 2200

The example plotted on Fig 3.3 shows, that most practical vent implementations operate in non-linear region, compromising performance at moderate to high power levels.

Screen Cursor

Screen cursor, incorporated in the “Frequency Domain” screen, has been implemented as a vertical line stretching across plotting area of the screen. The cursor is activated by clicking LEFT mouse button above the plotting area and cursor movements are controlled by “<-” and “->“ (left and right) cursor keyboard keys. Display fields, under “Legend”, in the bottom-left corner of the screen provide the exact frequency coordinate and value of all functions plotted.

For example - see Figure 3.3. To reposition the cursor line instantly, point the mouse arrow at the required section of the curve on the plotting area and click LEFT mouse button.