Chapter 17. Analog Nonlinear Measurements And Modeling

Non-linear Parameters Estimation using Volterra Series Expansion

The nonlinear parameters extraction process is quite involving. The general strategy of estimating non-linear distortion follows the works of A.J.M. Kaizer. and is based on **truncated Volterra series expansion**. In short, the strategy is as follows:

1. Measure linear (or small signal) parameters of the driver – $\mathbf{BL_0}$, $\mathbf{K_0}$, and $\mathbf{Le_0}$.
2. Measure loudspeaker first (F1), second (F2) and third (F3) harmonics over the low frequency region, with cone excursion close to Xmax.
3. "Curve Fit" the measured harmonics to the built-in model by manipulating coefficients of the BL(x) , K(x) and Le(x) equations:

$$\mathbf{BL(x) = BL_0 + b_1{*}x + b_2{*}\,x^2}$$
$$\mathbf{K(x) = K_0 + k_1{*}x + k_2{*}\,x^2}$$
$$\mathbf{Le(x) = Le_0 + l_1{*}x + l_2{*}\,x^2}$$

The $\mathbf{BL_0}$, $\mathbf{K_0}$, and $\mathbf{Le_0}$ are "force factor", stiffness (K = 1/Cms) and voice coil inductance known from the linear theory. The **b1,b1, k1,k2 and l1,l2** coefficients are manipulated during the curve fit process to obtain the best match with the measured harmonics over the desired frequency range. When the optimization process is finished, you should be able to obtain reasonable good fit between the measured and calculated data, indicating that calculated **b1,b1, k1,k2 and l1,l2** nonlinear coefficients represent real life distortions as close as possible – see the example Figure 17.1 below.

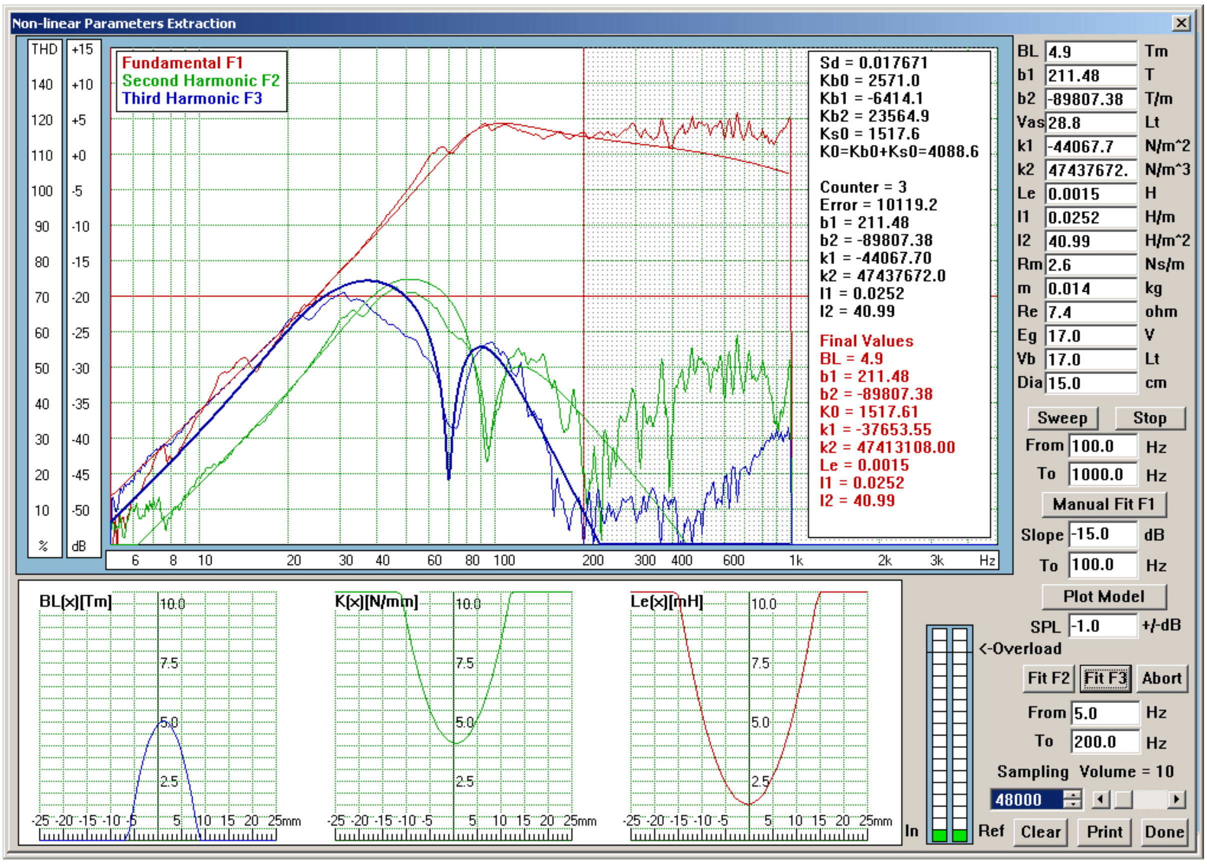


Figure 17.1 Estimation of non-linear coefficients for Volterra series expansion.

Comments On Performing The Measurements

There are many factors to consider before the actual measurements can be performed. Please remember, that the loudspeaker under test needs to be driven with a power level capable of producing high cone excursions (comparable with Xmax). The test case described in this manual uses an 8" driver, with power handling capabilities of 30W. The loudspeaker is mounted in an 17 liter enclosure and driven with approximately 20Watt of power.

1. All measurements are performed below 200Hz, so it is quite important to measure the loudspeaker correctly at such a low frequency range. The program will automatically generate, the sweep sine wave and track all three (fundamental, second harmonic and third harmonic) frequencies. However, you need to make sure, that reliable hardware and test environment for measuring frequencies down to 10Hz is provided.

   The measurement issues will be much reduced if you have an access to the anechoic chamber. The curves will be much smoother and will measure without the environment reflections and resonances. Basically, the issue to consider is the enclosure diffraction and overdriving the test microphone. The test microphone can be located at 1meter away from the loudspeaker. If the loudspeaker's efficiency is 85dB/W/m, the 20watt applied to the speaker will produce 98dB SPL. Most test microphones should be able to handle this pressure.

   If you do not have access the anechoic chamber, you may have to resort to the "close mike" techniques. The issue now is more complex. For instance, if you decide to keep the same power (20Watts) and bring the microphone closer to the speaker (say 12.5cm distance) the SPL pressure will produce 116dB at the microphone location. This is significantly more than discussed before and you need to make sure, that the microphone and the pre-amplifier can handle this conditions without introducing clipping distortions.

2. Voice coil inductance varies with frequency and the inductor is actually a "semi-inductor" type. The Volterra series expansion model assumes that the inductor is actually a "normal" inductance. Therefore, there will be a discrepancy between measured inductor value and the modeled inductor. The value of the inductor increases several times going towards low frequencies. On the example below, the inductance increase 6-fold from 20kHz down to 150Hz – see Figure 17.2 below (brown curve).

   This issue is quite critical and in our test example, the value of Le was increased from 0.8mH to 1.5mH, (2x) in order to account for the inductance variability below 200Hz.

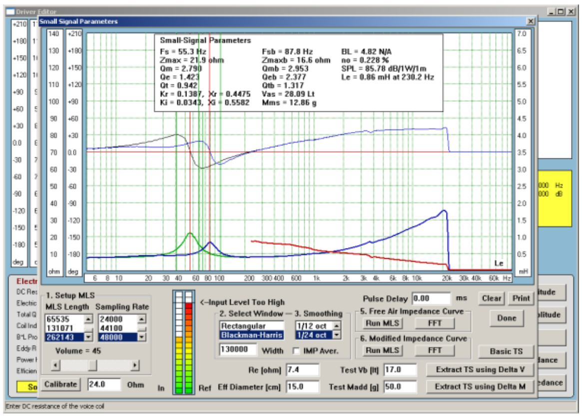


Figure 17.2 Measurement of linear parameters

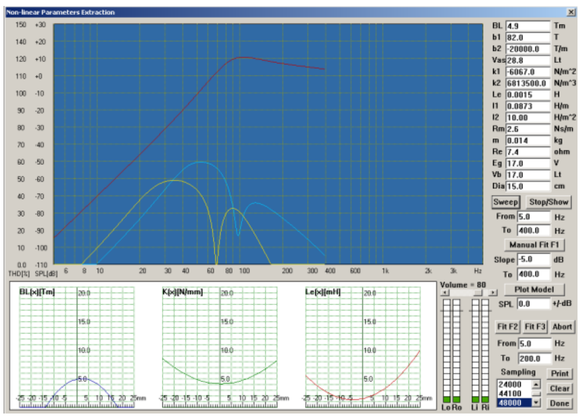


Figure 17.3 Default values for Volterra model.

3. Initial conditions for the "curve fitting" process are also important. The curve fitting mechanism is nothing more than looking for a minimum error between the measured data and the model. The mechanism will be therefore prone to typical "local-vs-global" minimum error issue. If you select unfortunate starting conditions for the optimizer, there is quite distinct possibility, that the optimizer will find "local" minimum, rather than "global" minimum. Starting conditions can be improved by inspecting the "default" curves plotted by pressing the "Plot default" button.

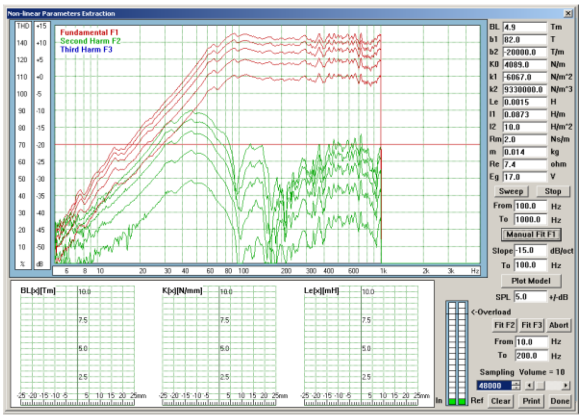


Figure 17.4 Second harmonics

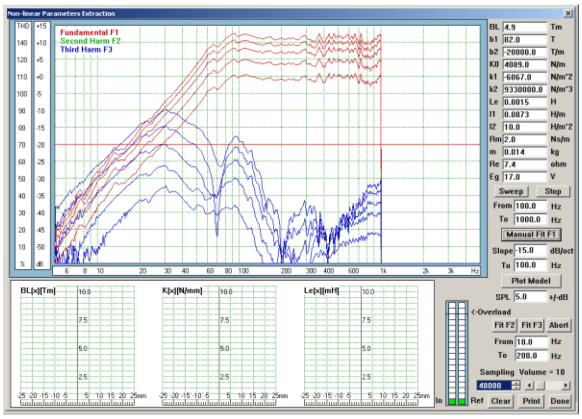


Figure 17.5 Third harmonics

4. The Volterra series expansion model is a truncated model, implemented for the purpose of evaluating the non-linear behavior of the loudspeaker. It is applicable to "weekly nonlinear" systems. In order to evaluate the required input voltage into the loudspeaker, we have plotted the **fundamental (red curve), second (green curve) and third (blue curve)** harmonics of our test loudspeaker for the following power levels top to bottom:

   34.0 Vpp……. 19 watts
   26.8 Vpp……. 11.8 watts
   20.0 Vpp……. 6.6 watts
   13.4 Vpp……. 3 watts
   7.8 Vpp………1 watt

The curves above 200Hz should be discarded, as the plots were taken in a very "bad" acoustic environment and room influence (modes) is clearly visible from the shape of the plots. However, the major characteristics of the second and third harmonics curves are clearly visible. The following can be observed:

   The shape of the second harmonic curves does not change very much with the input power level. The shape of the second harmonic curve is tied up to a straight line and the second harmonic distortion alone, relates mostly to "offset centers" or "skewed" curves and these parameters do not change significantly with the cone excursion. The overall level of the second harmonic plots will move up or down, depending on the input power, however the shape of the curve remains quite similar.

   The third harmonic curve behavior is somewhat different. The shape of the third harmonic curve is tied up in our model to the characteristics of a parabola. The shape of the third harmonic curve relates to how "wide" or "narrow" the parabola is, and the location where you are on the parabola curve. This ultimately changes the shape of the third harmonic curve as the cone excursion is changed.

   The accuracy of our truncated model will therefore depend on input power level to the loudspeaker. We recommend, that you should use input voltages, that produce cone deflections between 60-100% of Xmax. From the examples above, it is easy to observe, that the third harmonic curve begins to stabilize its shape above 6.6 watts of input power on a small, 30 watt rated loudspeaker. In this case, a 19 watts would be recommended.

5. Loudspeaker small-signal parameters change depending on the "break-in" duration. It is recommended to run the loudspeaker with 50-60% of rated power for at least 1 hour prior making the small-signal measurements. This would loosen up and stabilize the suspension, making it ready for the much longer non-linear tests. The danger here is, that the non-linear test runs for several minutes and by itself, is a form of "breaking-in" loudspeaker. If you start with "cool" loudspeaker for small-signal tests and then proceed right away into the non-linear tests, you effectively test the same speaker under somewhat different suspension state (stiffness).

6. The measurement system/environment may introduce its own characteristics at such a low frequencies. An example is shown below, where the measured SPL of the fundamental frequency does not follow +12dB/oct curve, expected for the loudspeaker placed in a sealed box.

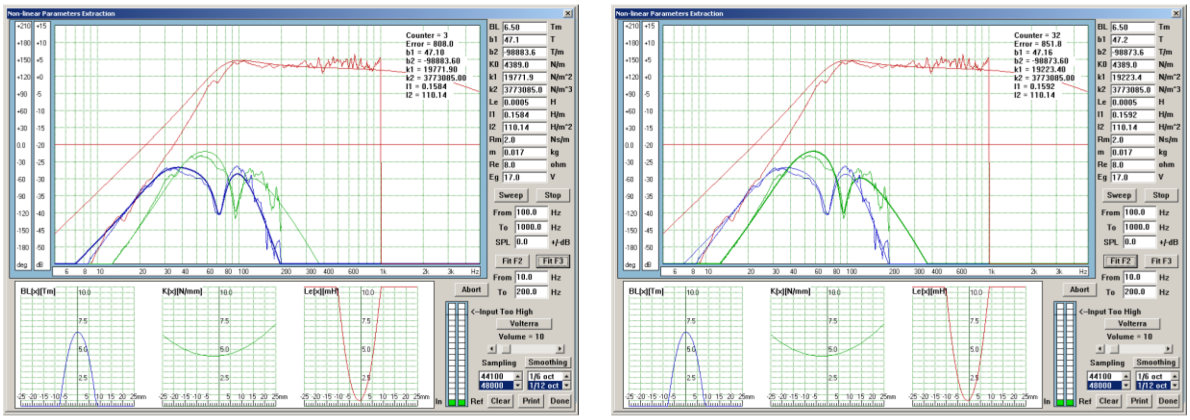


Figure 17.6 SPL mismatch on the fundamental frequency impacts curve fit for second and third harmonics.

In a drastic situation, the optimizer may produce erroneous results or can even “flip” the sign of the coefficient k2, if the test equipment does not have flat enough frequency response – see below. In this case, you need to review the total frequency response and environment of your testing process. If you are confident, that this is not the problem, you would manually correct the sign of the k2 (or b2) coefficient and attempt to re-optimize from perhaps slightly different starting conditions.

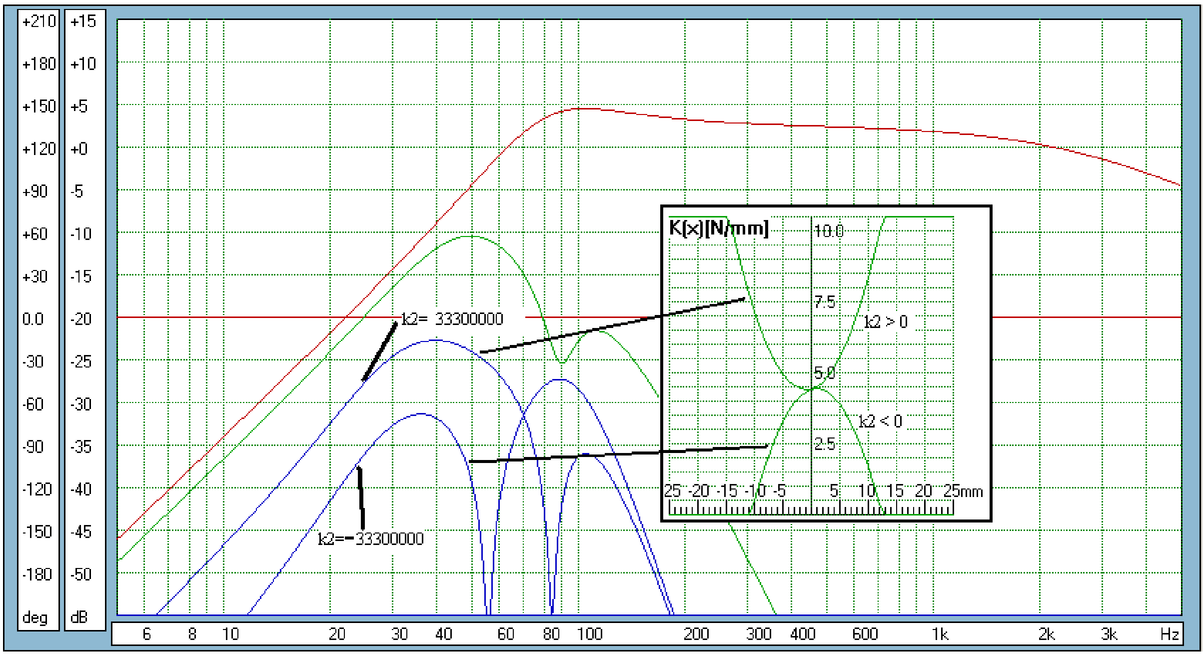


Figure 17.7 Effect of a negative k2 coefficient.

If you do have the problem with the total frequency response of the test equipment as shown above, you can remedy it to a significant degree by entering the required number of decibels you need to “lift” or “drop” the imported data, starting from the left-hand side of the screen (entered xx.x dB), and gradually tapering off and continuing to the nominated frequency entered right below the “Slope” field, where the modification no longer applies.

Making The Measurements

The block diagram below shows general idea for the test setup. The loudspeaker needs to be placed in a **sealed enclosure** of known volume. Stiffness of the enclosure will be added to the stiffness of the driver to give the total stiffness K0. The built-in non-linear model assumes that sealed box will be used in this test. Capturing all three harmonics over the required frequency range is accomplished in the following steps:

1. Set the sampling rate and output level using “**Sampling**” list box and “**Volume**” slider located at the bottom right-hand corner of the screen.

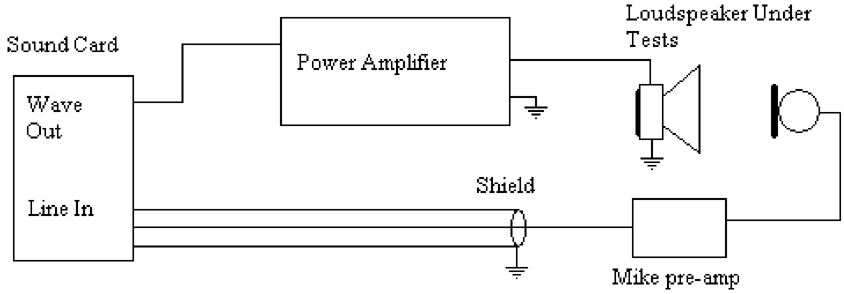


Figure 17.8 Suggested measurement setup.

2. Select sweep range by entering “**From**” and “**To**” frequencies under the “Sweep” button.
3. Commence the SPL measurements by pressing the “Sweep” button. You should see a picture similar to the one displayed below. You will notice, that the program tracks the fundamental frequency and second and third harmonics (and optionally 4th and 5$^{th)}$ and placed a red marker line at the measurement point – see below.

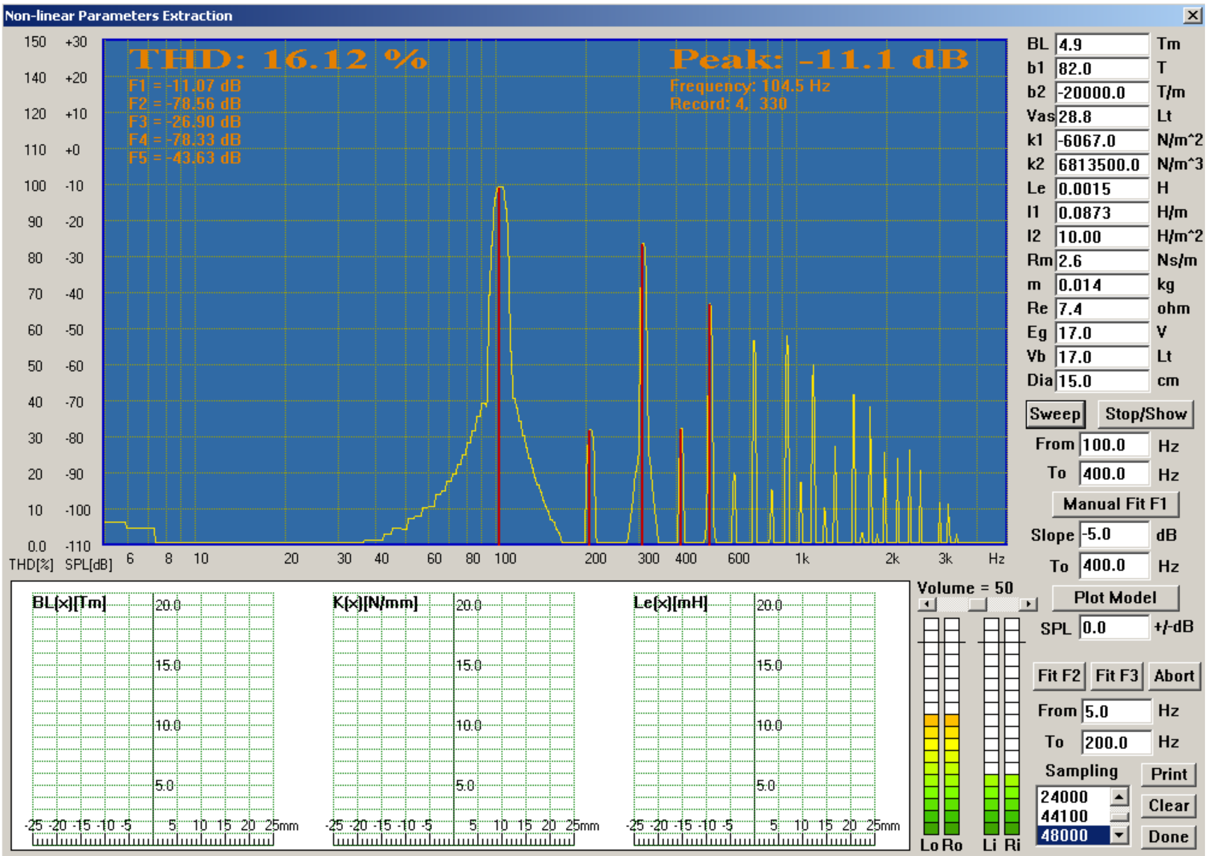


Figure 17.9 Fundamental = 104Hz, second harmonic = 208Hz and third harmonic = 312Hz (see RED markers)

4. It is important **NOT** to overload the input of the sound card (or any other component in your measurement chain) as this will add to the harmonics level distorting the measurements. Figure above is just an example of overloaded soundcard input just to show the harmonics tracking process.

5. When the sweep process is completed, the program will display the captured fundamental and harmonics – see below. Please note, that the SPL scale has automatically switched to 5dB/div. All three curves are stored in the current driver data file.

Please note, that the information just collected (plots of the fundamental and the first two harmonics) is of extreme value to the designer. You could easily stop here and review the plots for the purpose of evaluating the distortion level of your design. The measurement procedure described up to this point is applicable to any type of enclosure and driver combination. You can simply use this feature as a universal distortion analyzer.

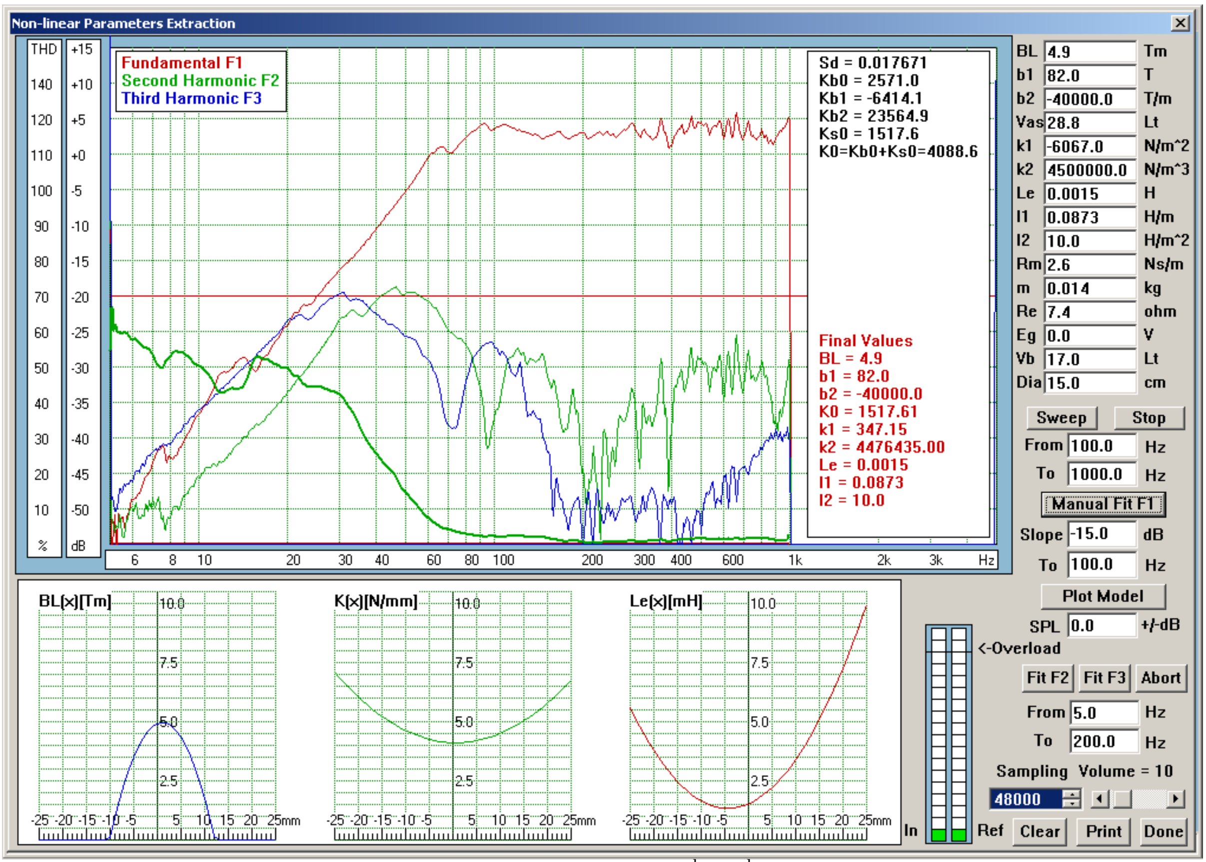


Figure 17.10. Captured fundamental and 2nd + 3rd harmonics and THD plot

6. You should now enter the parameters of the model into the fields provided:

Eg – is Vpp/2 (half of the peak-to-peak input voltage [V].
Re – is the DC resistance of the driver's voice coil [ohm].
m – is the vibrating mass (including air load) [kg].
Rm – is the mechanical resistance of the vibrating system [ohm].
Le – is the voice coil inductance at lower end of the frequency range [H]. Typically you would enter about 150% of the VC inductance at 100Hz.
Vas – is the loudspeaker Vas. [Lt]. Typically, the stiffness coefficient Ko = 1/(1/Cab + 1/Cms) would be used here, but because Vas is so widely used, we decided to use Vas and internally calculate Ko instead.
BL – is the motor force, as measured in linear section[Tm].
Vb – is the test box internal volume in liters [Lt].
Dia – is the loudspeaker diameter in centimeters [cm].

For now, you can use **k1, k2, b1, b2, l1 and l2** as provided default values. Enclosure non-linearity is built into the model using the following equations:

$$\mathbf{K_{box} = K_{b0} + K_{b1}{*}x + K_{b2}{*}x^2}$$

$$K_{b0} = \frac{\rho_0 c^2 Sd}{Vb}, \quad K_{b1} = K_{b0}(\gamma + 1.0)\frac{Sd}{Vb}, \quad K_{b2} = K_{b1}(\gamma + 2.0)\frac{S_d^2}{V_b^2}$$

$$K_{s0} = \frac{\rho_0 c^2 Sd}{Vas}$$

where: Sd = (Dia/2)*(Dia/2)*π, γ=1.40, Vas – is the equivalent compliance volume of the driver, Vb - is the enclosure volume, $\rho_0 = 1.18 kg/m^3$.

The Kb0, Kb1 and Kb2 parameters are automatically calculated and displayed on the screen.

7. The above parameters should allow you to plot the SPL of the fundamental frequency. At this stage, the goal is to manually provide reasonably good match between the plots of the fundamental SPL measured on your driver (red wiggly curve) and the fundamental SPL of the built-in model (smooth red curve) over the frequency range defined under the "Fit F2" and "Fit F3" buttons. This process could be automated, but there are some advantages by doing it manually – you can simply "brute-force" the parameters as you want them. The process is rather simple. Press **"Manual Fit F1"** button and adjust data under this button for "Slope" and "To" frequency to get close second order slope matching. Secondly, you may need to move the model curves UP/DOWN using the "SPL" data (under the "Plot Model" button) depending on the amount of system attenuation and test voltage applied to the speaker.

   The **thick green curve** displayed on the screen is a **"semi-THD"** plot. It is called "semi" as the formula for calculating it only includes the second and third harmonics. It is however useful in the analysis.

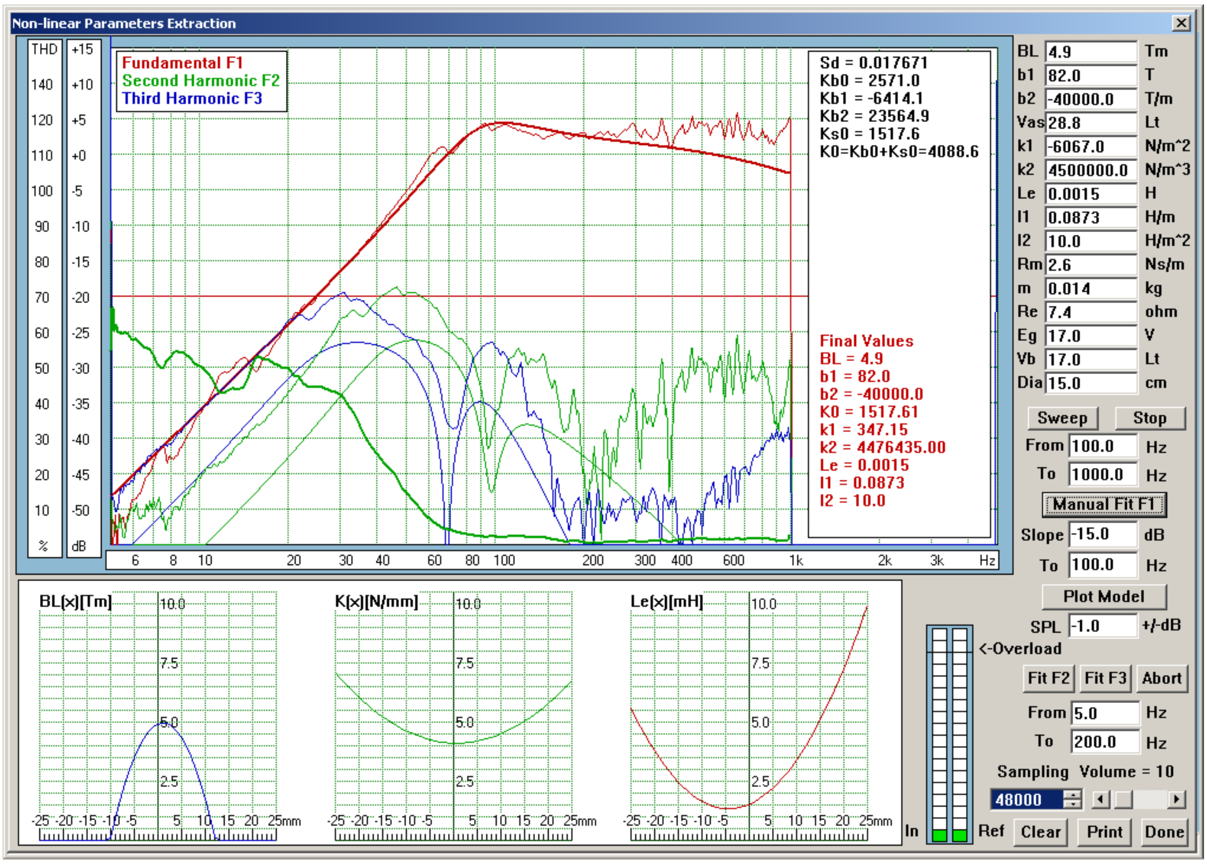


Figure 17.11 Matching SPL of the first harmonic from 5 Hz to 200 Hz – thick red curve

8. The final two steps in the process are "curve fitting" of the model into the measured second and third harmonics. At this stage, please enter the desired frequency range for the optimizer - the data fields are located under "Fit F2", "Fit F3" buttons**. It is recommended to keep the 5Hz – 200Hz default frequency range.**
9. Now press **"Fit F2"** button. The optimizer will perform two consecutive optimizations for the second harmonic. The only model parameters that are manipulated at this stage are: **k1, b1 and l1**.
10. Next, press **"Fit F3"** button to fit model into the third harmonic. The only model parameters that are manipulated at this stage are: **k2, b2 and l2**.
11. **It is strongly recommended to <u>repeat steps 9 and 10 several times in this order</u>. This is because results of the second order optimizations affect the third order optimizations. You should see it very clearly, that further attempts to fit the curves do not produce any significant reduction in error and this is the time to conclude the process. It is often beneficial to disturb Rm, m, BL, Vas, Dia parameters by 5-10% to see the reaction of the curve fitting process.**

The results of the curve fitting process on our example speaker are shown below.

BLo = 4.9 [Tm] **b1 = 211.48 [T]** **b2 = -89720.87 [T/m]**

Kso = Ko – Kbo = 4088.6 – 2571.0 = 1517.61 [N/m]
k1s = k1 – kb1 = -44067.7 - (-6415.1) = -37653.55 [N/$m^{2]}$
k2s = k2 – kb2 = 48005816.0 - 23564.9 – 47981252.0 [N/m^3]

Leo = 0.0015 [H]   l1 = 0.0252 [H/m]   l2 = 41.09 [H/m^2]

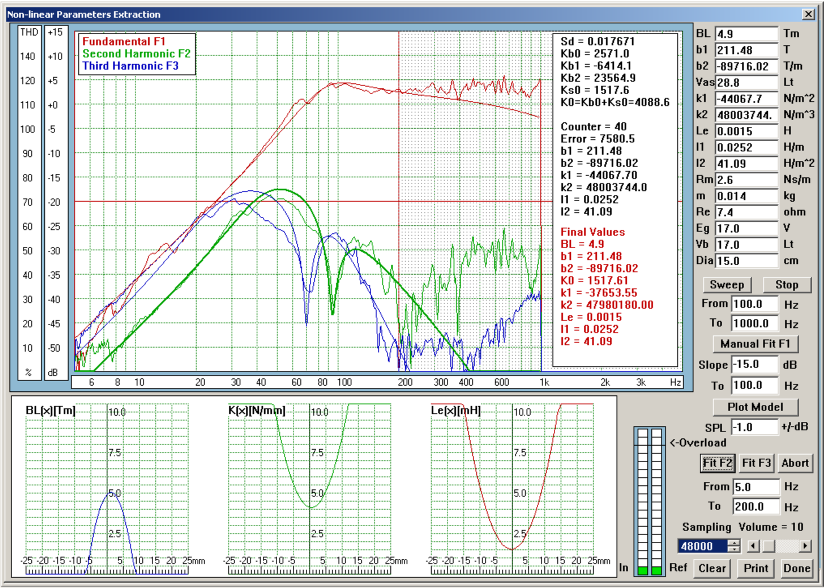


Figure 17.12 Result of “curve fitting” for F2

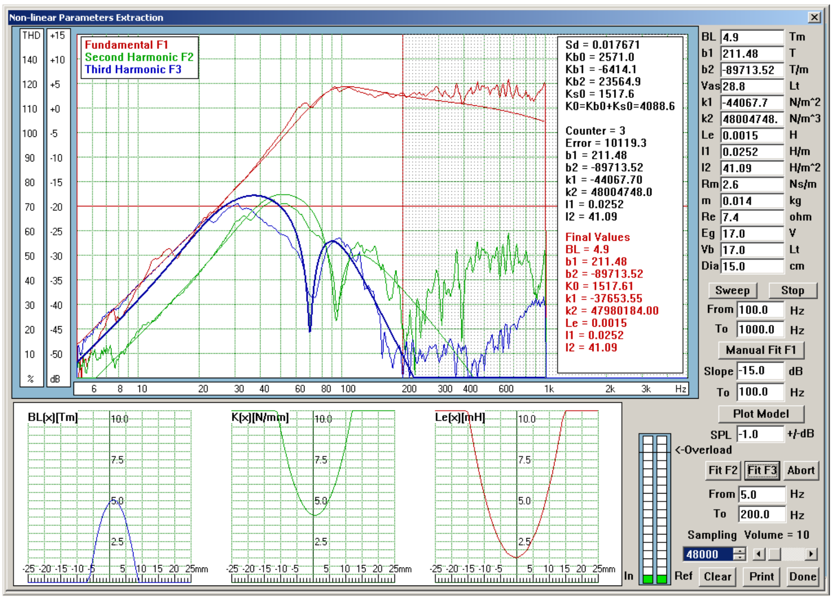


Figure 17.13 Result of “curve fitting” for F3

As mentioned in the introduction, the measurement process describe above is complex. In many instances you can split the whole operation into two steps:

1. Capturing fundamental, second and third harmonics for the purpose of evaluating the nonlinear distortion of the design. This process is quite simple, as all the necessary functions are highly automated and only require you to frequency sweep the system under test. The process of generating the sweep signal and tracking and capturing of the harmonics is fully automated.
2. Extracting coefficients of the BL(x) , Ks(x) and Le(x) for the purpose of further modeling of the system. Please be aware, the extracted coefficients are largely dependant on the accuracy of the measurements. Also, all three coefficients represent parabolas. This type of curve is a truncated approximation of the real-life characteristics of the loudspeaker non-linearity and will characterize some speakers better than others

Non-linear Parameters Estimation Using Differential Equations Method

Preferably, the test should be conducted in anechoic chamber or at least in "half-space" acoustic environment. Loudspeaker under test is mounted on a test fixture in "free air". The fixture does not provide any enclosure, it only serves to mechanically support the driver. The loudspeaker will be driven quite hard – up to 20Vpeak, so special precautions need to be taken. For instance, if the loudspeaker's efficiency is 85dB/W/m, and 20watt applied to the speaker will produce 98dB SPL at 1meter. Most test microphones should be able to handle this pressure.

If you do not have access the anechoic chamber, you may have to resort to the "close mike" techniques. The issue now is more complex. For instance, if you decide to keep the same power (20Watts) and bring the microphone closer to the speaker (say 12.5cm distance) the SPL pressure will produce 116dB at the microphone location. This is significantly more than discussed before and you need to make sure, that the microphone and the pre-amplifier can handle this conditions without introducing clipping distortions. The basic test setup is shown below and captured loudspeaker response for linear (red) and nonlinear operation (blue) is shown on Figure 17.15.

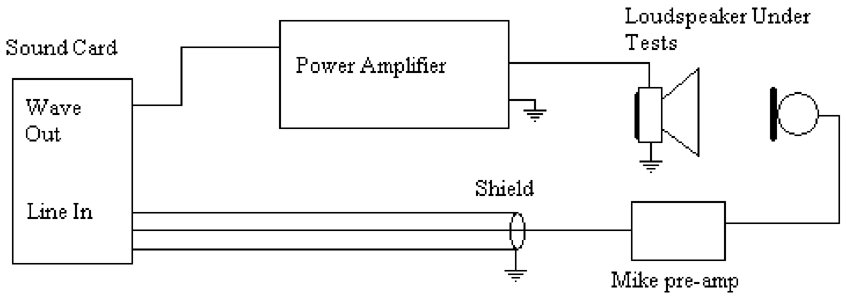


Figure 17.14. Test setup for Differential Equations Method.

Capture Issues to Consider

The essence of this measurement method is to excite the loudspeaker with sufficiently high voltage, so that it can be driven into the nonlinear region of BL(x) and Cas(x), record the cone excursion and then curve fit model parameters to the measured data in time domain.

The output of the sound card will deliver a tone burst of 10-20 periods of sine-wave at the frequency selected from the **"Generate Tone Burst"** group of controls. In response to the burst, we are attempting to capture the steady-state response of the loudspeaker, free of any transient distortions. There may be several reasons why you may want to cut out the initial part of the captured burst. For one, the loudspeaker needs to finish the initial ringing due to applications of large input voltage burst. Secondly, the sound card may produce some voltage fluctuations on the initial part of very first buffer output due to its internal workings – this was our observation when using Delta 410 card from M-Audio.

To overcome the above issues, you may need to uncheck the **"Save Linear Result"** and **"Save Nonlinear Result"** check boxes (to prevent the test results from being automatically saved to disk) and generate tone burst several times observing captured signal on the screen. The aim is to **see several periods** of the chosen frequency, with the **<u>same amplitude in each period</u>** – see Figure 17.15. The captured signal will be aligned to the left side of the screen, ready for curve fitting process. The alignment will be guided by the negative-to-positive voltage transitions of the captured signal. Therefore, it is important, that we cut out all "false" or undesirable transitions. On Figure 17.15, we have captured 2.5 periods of 40Hz distorted sine-wave and 8 initial "bad" transitions were skipped. It is observable, that linear response AND nonlinear response of the loudspeaker are correctly captured and automatically aligned on the screen.

The actual output voltage generated by the sound card can be adjusted using "Volume" slider in the **"Generate Tone Burst"** group of controls. We would recommend to set the card output between 75-90% and use your external power amplifier volume control to adjust the actual voltage measured across the loudspeaker. You will need to measure the **<u>peak value</u>** of the actual voltage across the loudspeaker for linear test and nonlinear test. These voltages will be used by the program to perform necessary internal calibrations. For entering the voltage amplitude of small-signal (or linear) operation, please use **"Linear Speaker Vin"** field – default is 2.50volts. For entering the voltage amplitude of large-signal (or non-linear) operation, please use **"Nonlinear Speaker Vin"** field – default is 20.00volts.

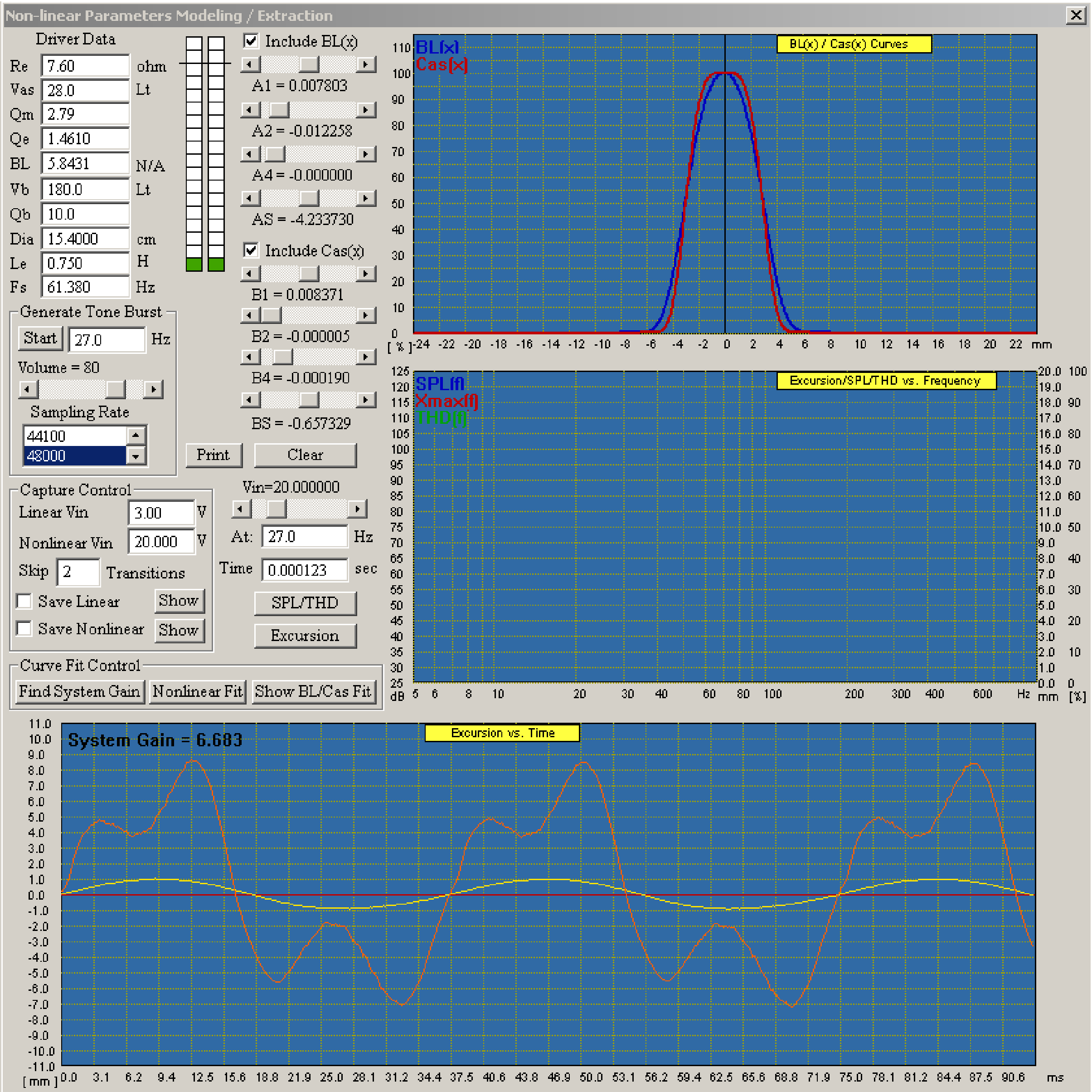


Figure 17.15. Captured loudspeaker response for linear (yellow) and nonlinear operation (orange).

Sampling rate for this test is fixed at 24kHz. It can not be changed from the screen list box. Loudspeaker parameters required by this screen are basic “free air” T/S data. ALL T/S fields included on this screen need to be supplied with correct data. The default starting data for BL(x) and Cas(x) exponential fit curves is shown below.

$$BL(x) = BL_0 e^{A4(x-S)^4 + A3(x-S)^3 + A2(x-S)^2 + A1(x-S)}$$

A4 = -0.0005 Wide (low) or narrow (high) of the curve as it falls to Y=0
A2 = -0.05 Flat (low) or Peaked (high) around X=0
A1 = 0.0 Asymmetry about X=0
S = 0.0 Shift of the entire curve about the X=0

$$Cms(x) = Cms_0 e^{A4(x-S)^4 + A3(x-S)^3 + A2(x-S)^2 + A1(x-S)}$$

A4 = -0.0005 Wide (low) or narrow (high) of the curve as it falls to Y=0
A2 = -0.04 Flat (low) or Peaked (high) around X=0
A1 = 0.0 Asymmetry about X=0
S = 0.0 Shift of the entire curve about the X=0

The captured data for linear and non-linear operation of the loudspeaker is automatically saved into the hard disk the way you see it on the screen. You can disable saving process for liner and non-linear data by checking or un-checking the two boxes located in "Capture Control" group. You can only "check" one of the boxes at a time, but you can uncheck both of them. The program makes sure, that you can not have them both checked at the same time. The corresponding **"Show"** buttons re-load the saved data and display it on the screen. Assuming, that you have captured the measurement results for your speaker, you may load them both onto the screen. Looking at the shape of captured example non-linear response, it is easy to observe, that the curve is asymmetrical around 0.0V. This would lead us to expect some asymmetrical shape in BL(x) , Cas(x) or both.

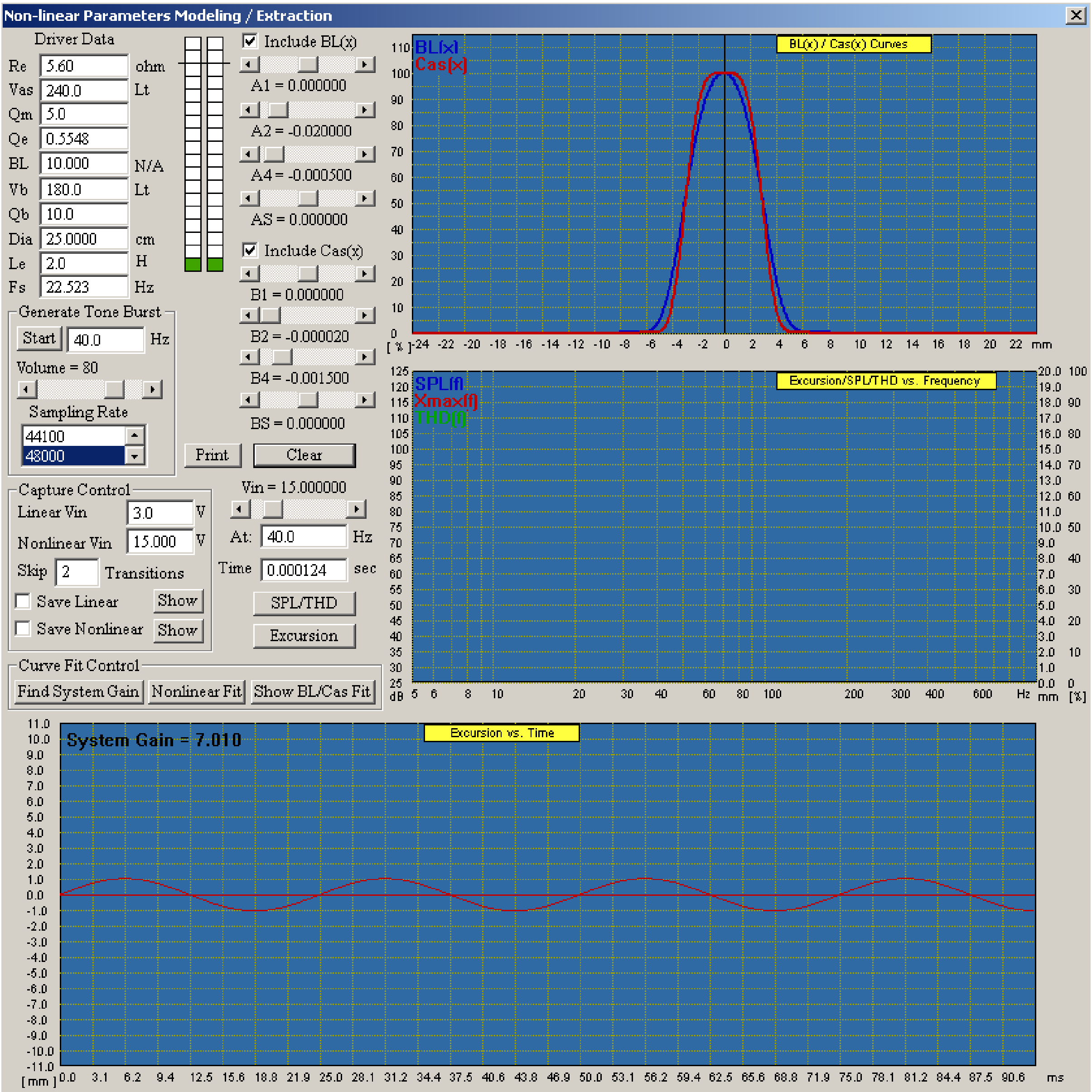


Figure 17.16. Result of "Find System Gain" operation – your system gain is determined.

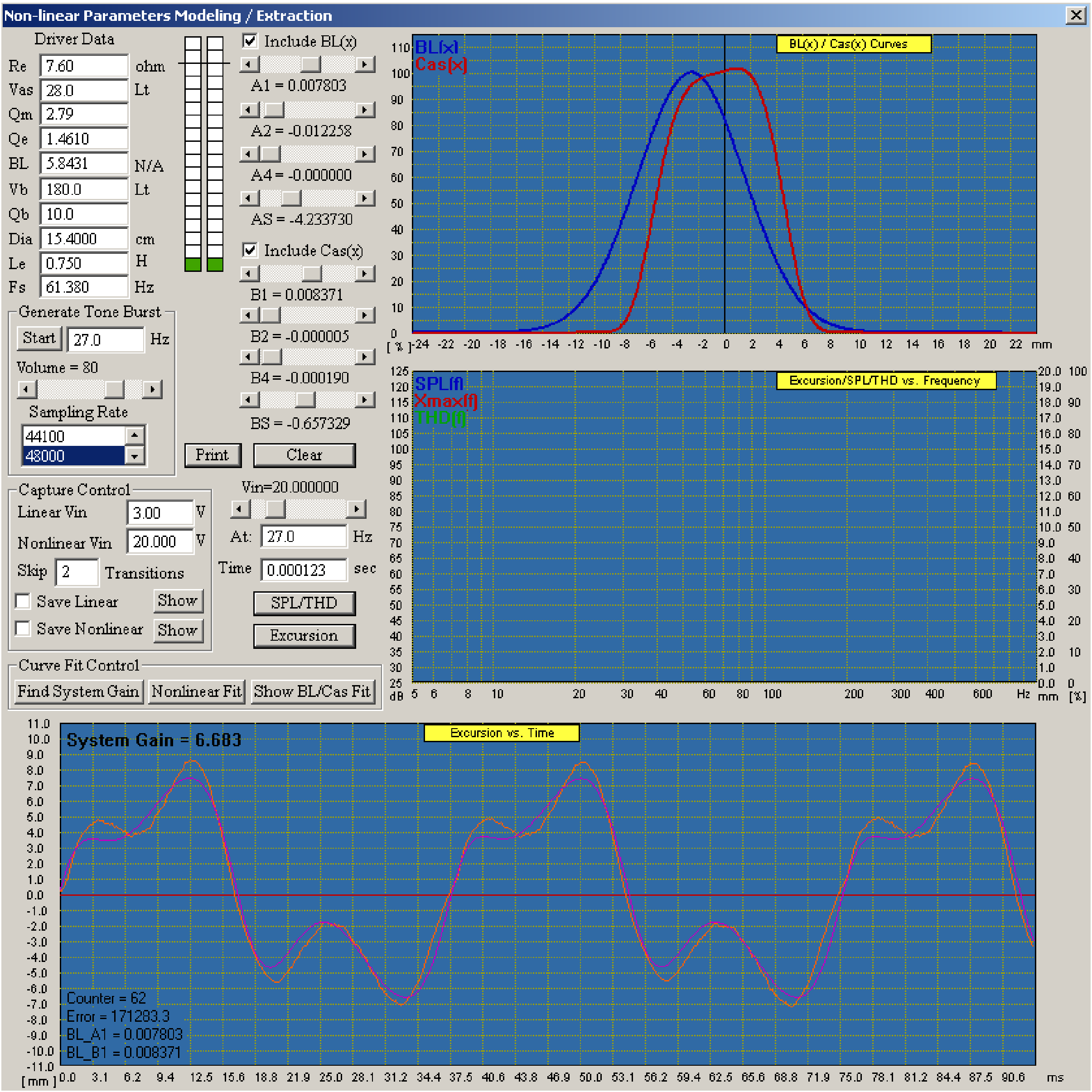
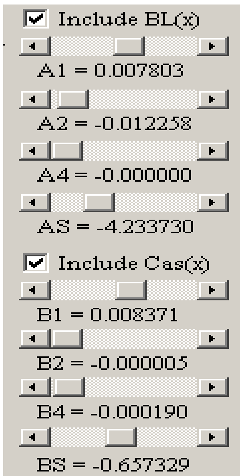


Figure 17.17. Result of "Nonlinear Fit" – driver parameters are found.

When the curve fitting process is completed, you should see both curves BL(x) and Cas(x) displayed on the screen – see Figure 17.18 as an example. Optimized valued for BL(x) curve (the A-coefficients) and Cas(x) curve (the B-coefficients) are shown on the right-side of the screen and the sliders are automatically set to those values.



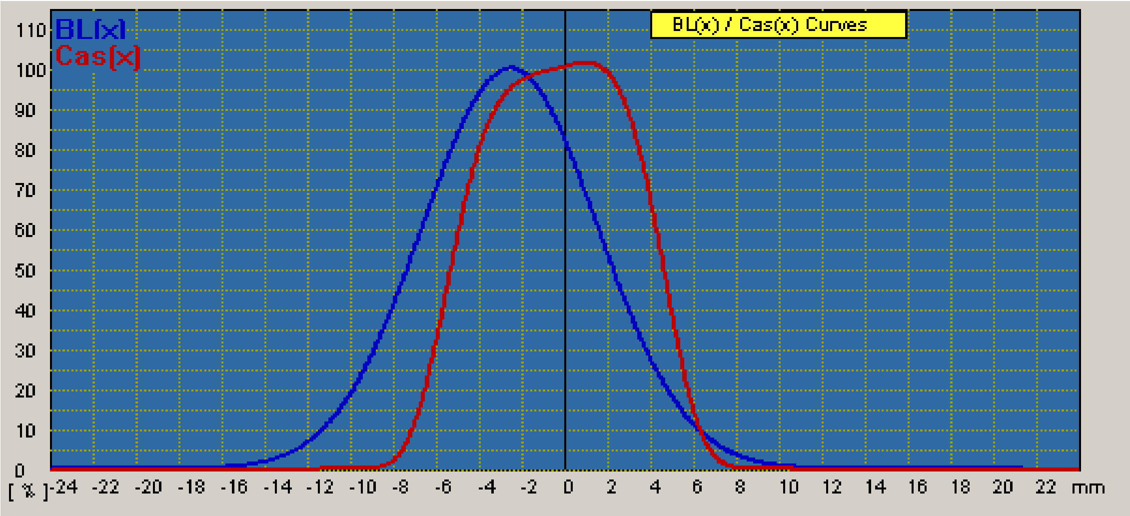


Figure 17.18. Optimized BL(x) and Cas(x) curves.

Other controls on this screen will be operational when you load all required T/S parameters first and include:

1. **"SPL" -** button used for plotting frequency response and cone excursion of the driver in sealed test enclosure.
2. **"THD" -** button used for plotting THD of the driver in sealed test enclosure.
3. **"Excursion" -** button used for plotting cone excursion of the driver in sealed test enclosure.
4. **"Spectrum" -** button used for plotting spectrum response of the driver in sealed test enclosure.
5. **"At" xxxx "Hz" -** data field used for entering frequency at which cone excursion is modeled.
6. **"Time" xxxx "sec**" - data field used for entering time step for Runge-Kutta algorithm.

7. **BL(x) Shape**
   A4 = Wide (low) or narrow (high) of the curve as it falls to Y=0
   A2 = Flat (low) or Peaked (high) around X=0
   A1 = Asymmetry about X=0
   S = Shift of the entire curve about the X=0

8. **Cas(x) Shape**
   B4 = Wide (low) or narrow (high) of the curve as it falls to Y=0
   B2 = Flat (low) or Peaked (high) around X=0
   B1 = Asymmetry about X=0
   S = Shift of the entire curve about the X=0

9. **"Include Cas(x)"** – check box to include compliance effects in model.
10. **"Include BL(x)"** – check box to include magnetic motor in model.
11. **"Vin"** – slider to adjust input voltage across models' terminals.
12. **"Clear"** – button to clear the screen.
13. **"Driver Data" –** 10 data fields for Small-Thiele parameters.

Measurement method

1. Enter Driver's T/S parameters, or load fully edited driver file.

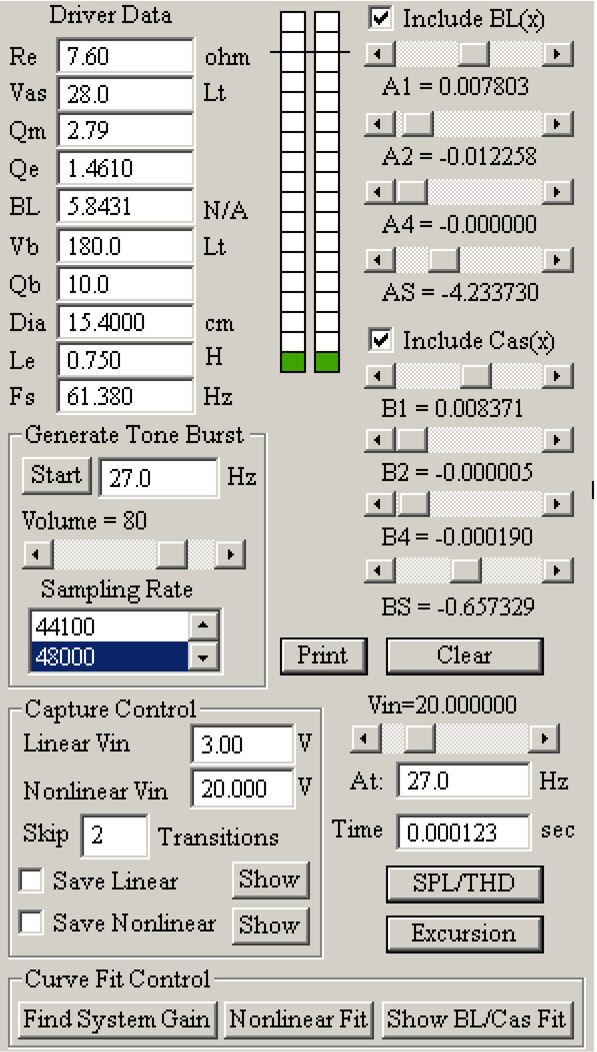
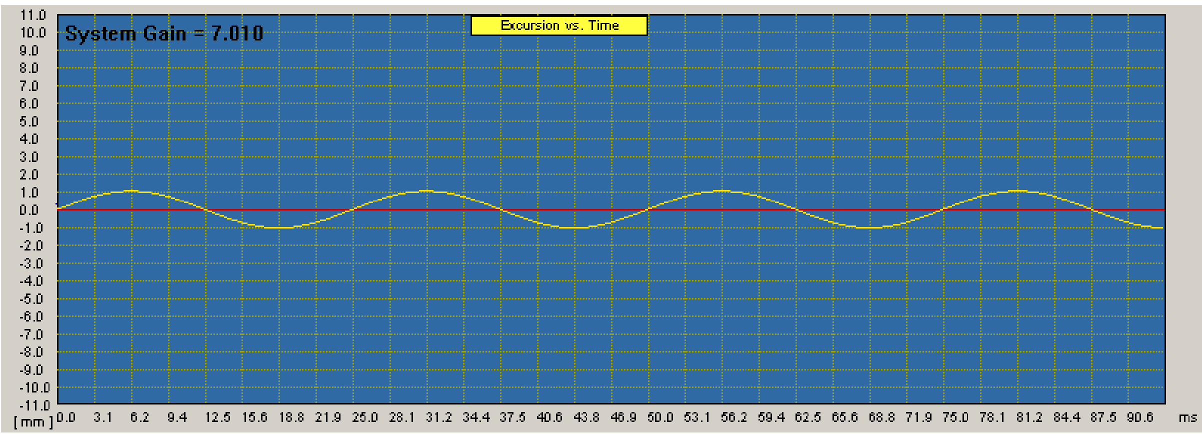


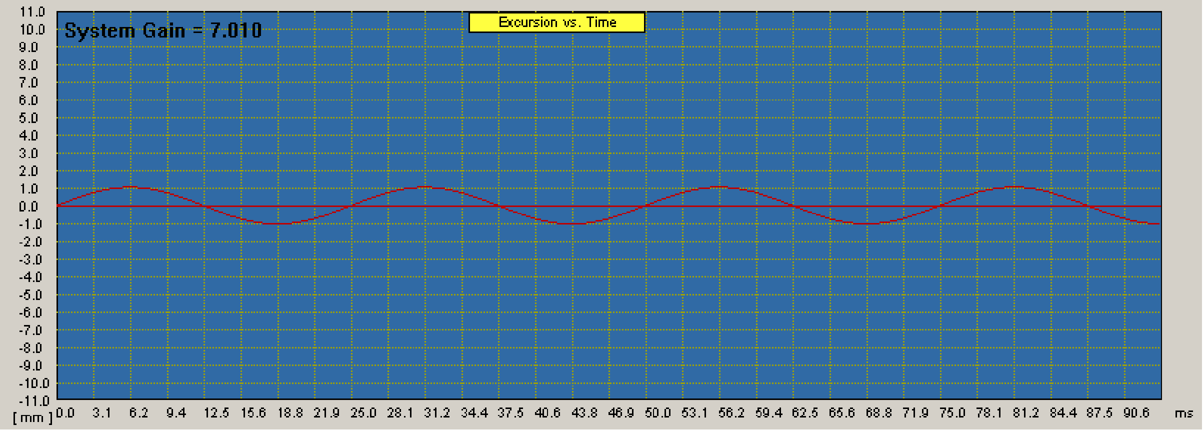
Fig 17.19. Driver T/S parameters entered.

Please note, that this driver was mounted on a baffle, and Vb was entered as larger value: Vb=180Lt.

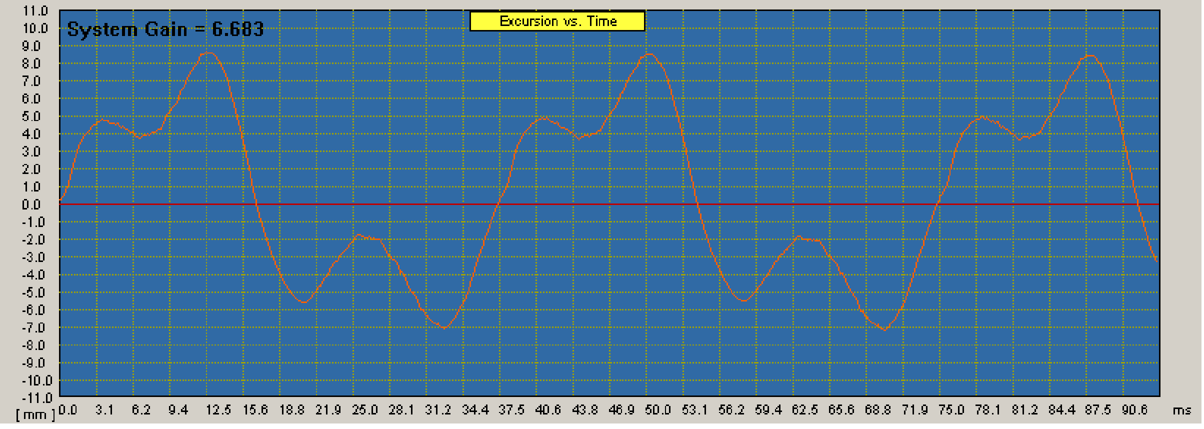
1. Select Tone Burst frequency – say 27Hz.
2. Select Volume – say 50 level.
3. Set output voltage across loudspeaker terminals to 3.00Volt PEAK.
4. Enter the same 3.000 into "Linear Vin" data field.
5. Press "Start" button and play the burst several times to see, if you can capture 2-3 periods of good, clean sinewave – see figure below:



6. Check “Save Linear Result” and press “Start” button to finally save the result.
7. Press “Find System Gain” button to calibrate internal data. System Gain is saved in Preferences file.

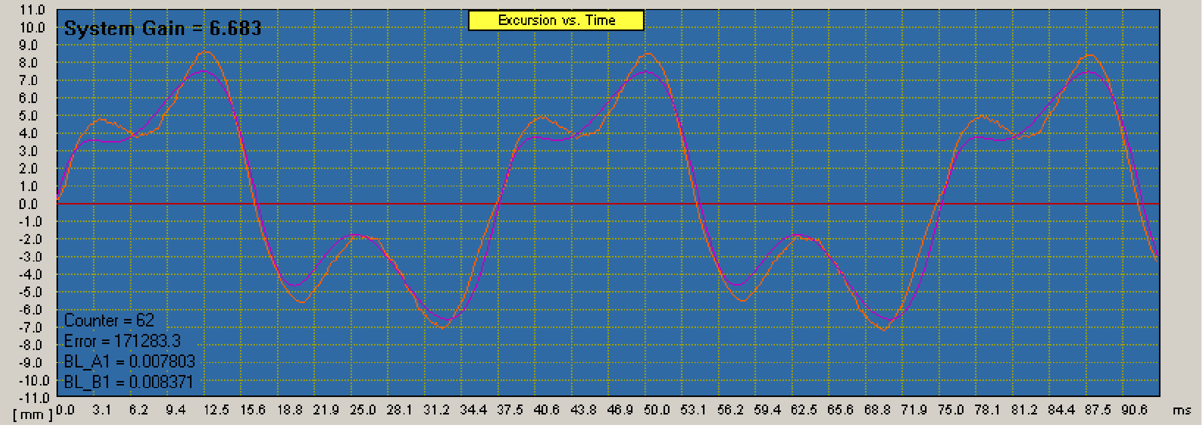


8. Increase voltage across loudspeaker terminals to driver the speaker into distortion. You should see something like figure below:

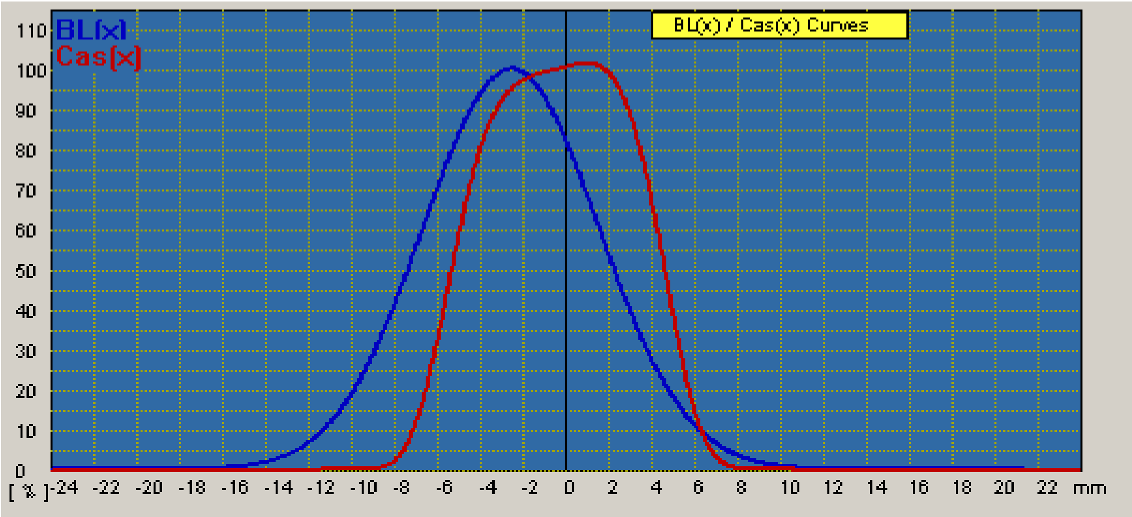


9. Enter the same increased voltage (say 20.0) into “Nonlinear Vin” data field.
10. Press “Start” button and play the burst several times to see, if you can capture 2-3 periods of good, clean sinewave – see figure above
11. Check “Save Nonlinear Result” and press “Start” button to finally save the result.
12. Press “Non-linear Fit” button.

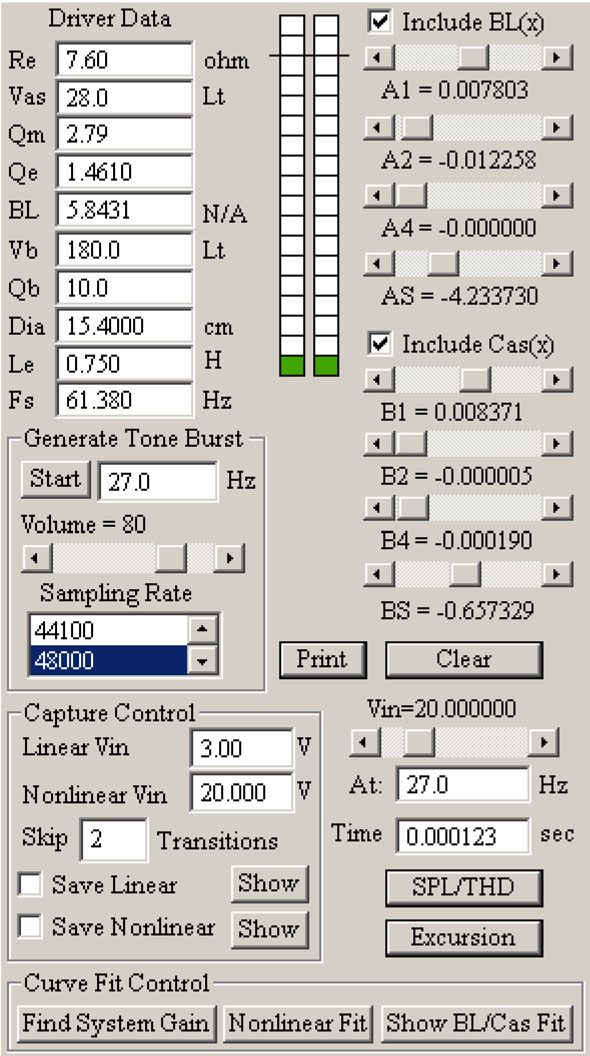
Now, the “curve fitting” algorithm make a number of attempts to re-adjust all As’ and Bs’ coefficients to minimize the error between measured, distorted waveform and the simulated one. You should obtain figure similar to the one below:



13. BL(x) and Cas(x) curves – see below – are shown automatically.



14. All Non-linear A's and B's are calculated.



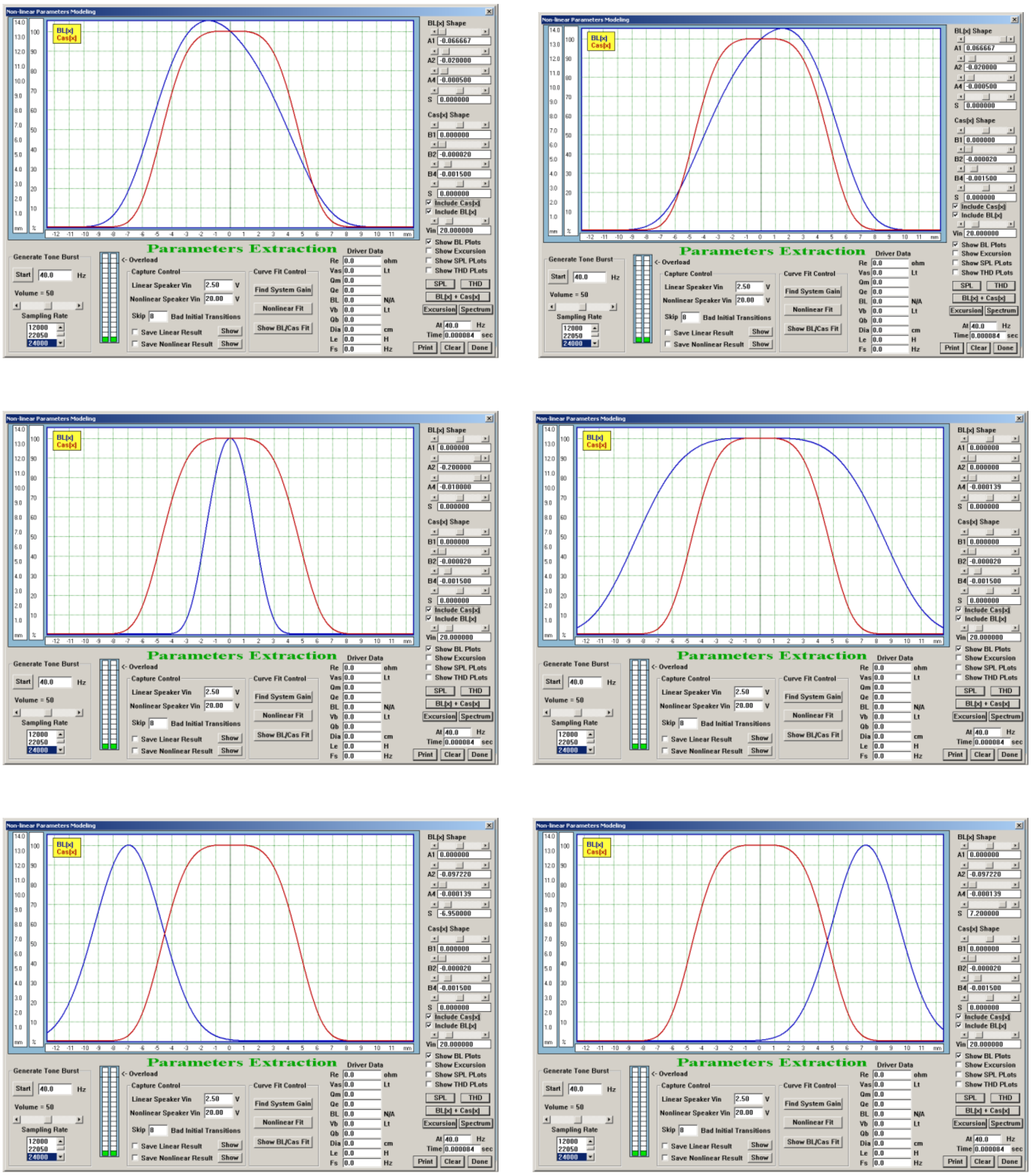


Figure 17.20. Effects of different A1, A2, A4 and S slider positions on BL(x) curve

Modeling Second Order Systems

The "**Non-Linear Parameters Modeling**" screen can also be used to predict SP/ THD/ Cone Excursion of a loudspeaker mounted in a sealed enclosure.

1. Enter **"Driver Data**" T/S parameters
2. Select **"Vin**" voltage.
3. Select test frequency "**At:" in Hz**
4. Press **SPL / THD** button for appropriate plots**.**

Shown below, is an example of a loudspeaker in a 180lt sealed enclosure modeled with the given BL(x) and Cas(x) parameters. Please not the voice coil "jump" at 80Hz also discussed on page 4.62 of the manual.

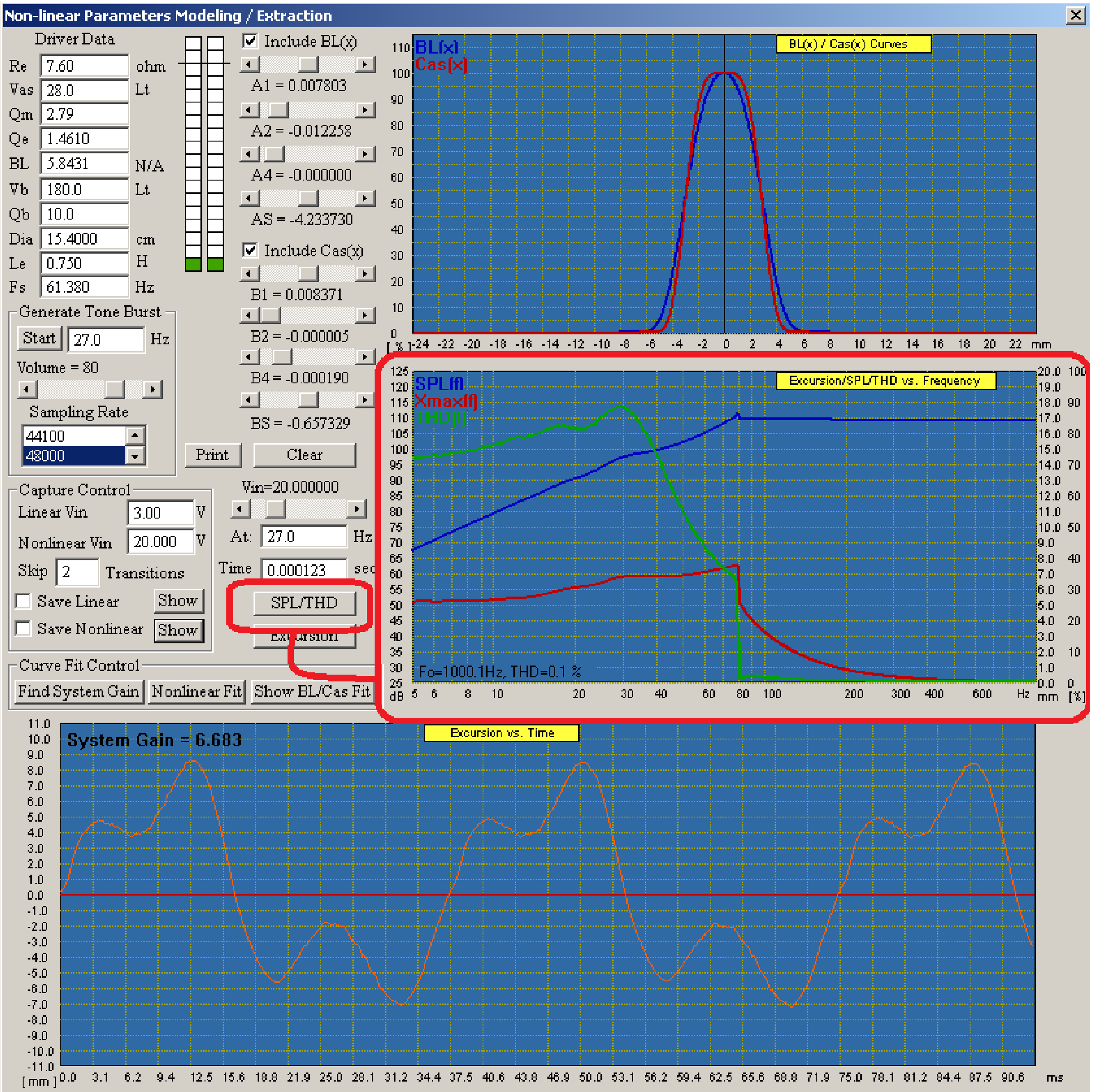


Figure 17.21. Example of modeling second order (sealed box) systems.