Modelling of Cone Break-Up in Electro-dynamic Loudspeakers

By Bohdan Raczynski

1. Background

A popular method of measuring loudspeaker sound pressure response in-room is based on a simple technique preventing significant room interaction with the measured results. This technique involves measuring the SPL using "close-mike" approach, and then simply adding the diffraction effects of the enclosure, modelled at 1meter distance. The combined response is supposed to be equivalent to the standard 1m-distance-type of measured sound pressure.

From my practical experience, I have always had difficulties matching the sound pressure response modelled in the above way with the SPL measured at 1meter distance. Things looked pretty good over the pistonic-range of the loudspeaker's frequencies , but the SPL curves deviated apart for higher frequencies, above the pistonic range.

Graphs presented below were generated using two SPL measurement techniques: analogue gated technique, and MLS windowed technique. The results are very close to each other for both the analogue and digital measurement techniques, and indicate clearly, that adding a simple diffraction curve to the close-mike curves would not produce the curves measured at 1meter distance, beyond the pistonic range.

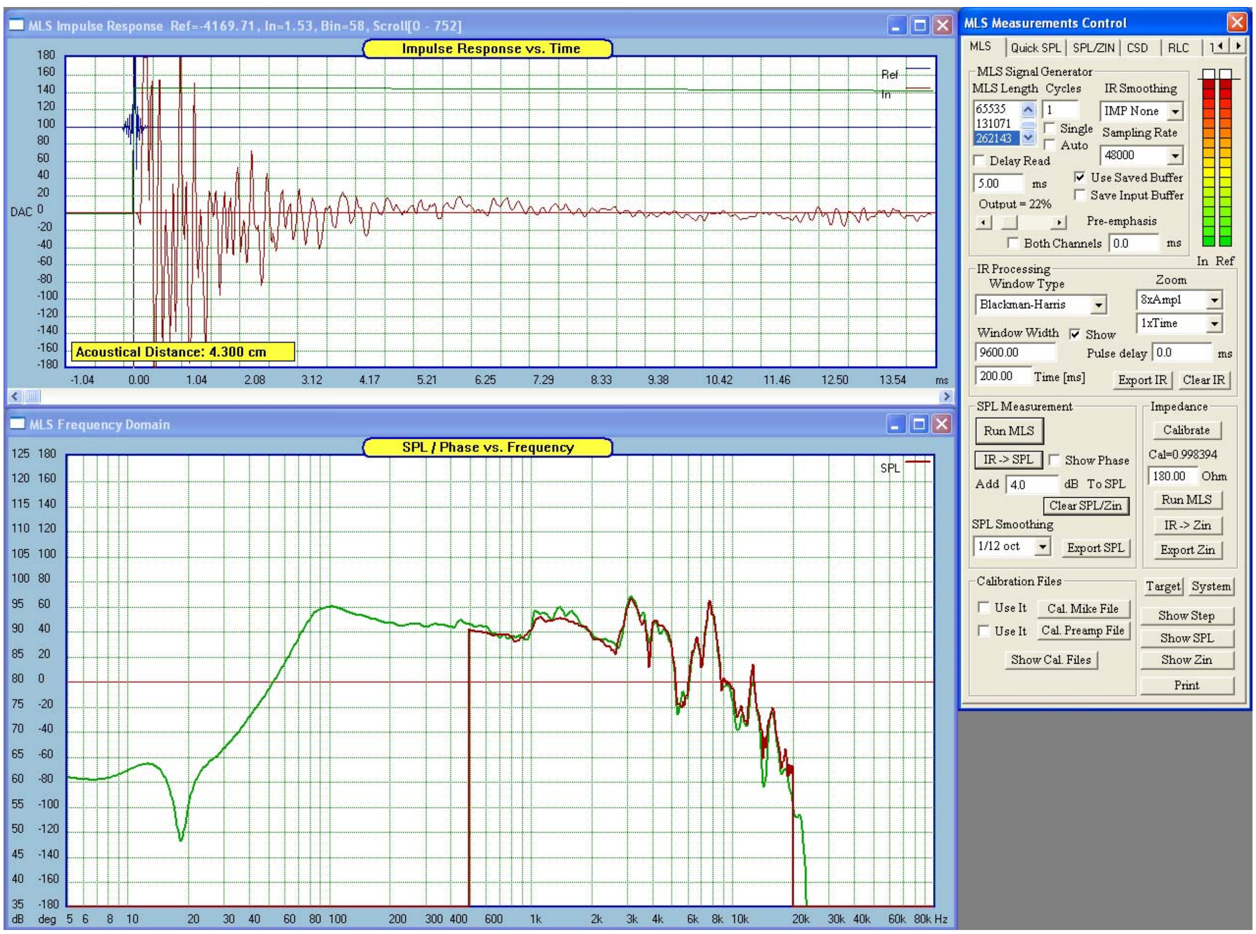


Figure 1. Close-mike SPL measurements. Green – MLS @5mm, Red – Gated @5mm

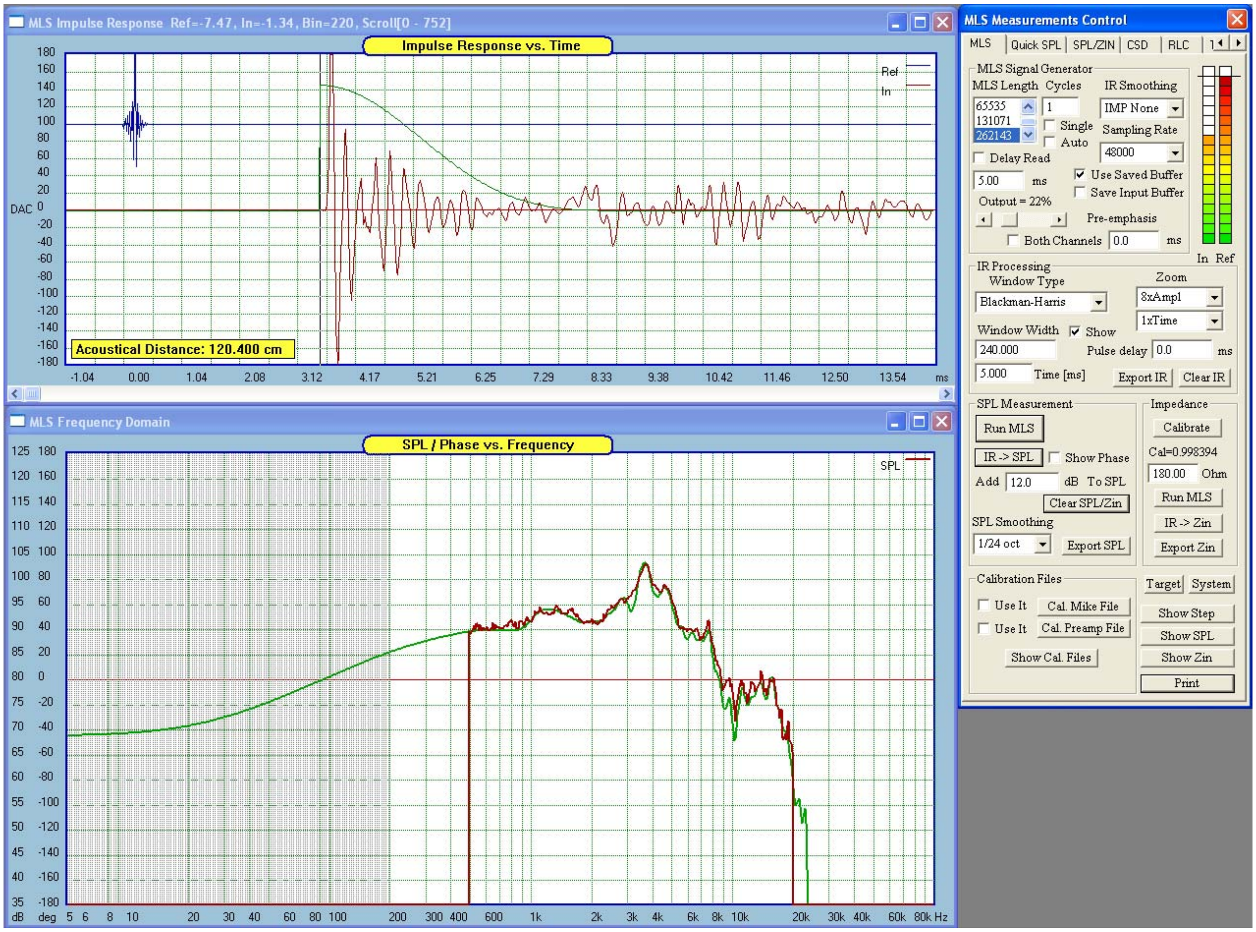


Figure 2. 1-meter distance SPL measurements. Green – MLS @1m, Red – Gated @1m

The most likely source of discrepancies was the exclusion of cone break-up effects above the pistonic-range of driver's operation.

Cone break up occurs when a loudspeaker diaphragm is driven at such a high frequency that modal (standing wave) effects form on the diaphragm. It is then no longer moving as a rigid body, making its radiation more complicated

To avoid cone break-up a loudspeaker system will use a range of two (or more) loudspeakers of different sizes to handle different parts of the audio bandwidth. The signal is "switched" between the drivers using an electronic filter network - the crossover. This approach ensures that the directional response of the total loudspeaker system is more constant over the operating bandwidth than would be possible for a single driver (given the expected polar response patterns).

2. FEM Model

Many dynamic problems in sound reproduction field involve combined motion of fluid (air) and structure (cone). This is commonly termed as "coupled system", because, in general terms, there may be some two-way interaction between the fluid and structure. Acoustic vibrations would fall into a category of "problems of long duration with limited fluid displacement". Accordingly to Betess[1], this class of problems can be approached from a specific angle.

When modelling sound reproduction aspects, on the fluid (air) side, we conclude, that the loudspeaker's cone radiates into a free space, therefore compression forces on the air may be negligible, as there is no enclosed cavity surrounding the air. Also, there will be no surface acoustic waves included in the model – there are just two major components of the model – vibrating cone and a layer of air in front of the cone.

Given the above, the "coupled system" problem can be reduced to an "added mass" problem. Consequently, the structural mass matrix M[][] is augmented by the mass of the air in the immediate vicinity of the cone. Typically, the air-load on the cone is within 10% of the total dynamic mass of the vibrating system. This approach follows the standard air-load model developed for lumped (Thiele/Small) loudspeaker models. Additional air mass can be calculated from the following formula:

$$m = \frac{8}{3}\rho_0 a^3$$ where "a" is the diaphragm radius, and ρ_0 is the air density.

In summary, there will be two FEM mesh models: one representing vibrating system of the loudspeaker and the second one representing the air in front of the cone. Vibrating cone will be described by it's geometrical dimensions and also by a whole host of material properties used to assemble the vibrating system. In general, this requirement will present significant difficulties, as the material properties used to assemble the loudspeakers are not widely available. Unfortunately, this requirements alone, will make the analysis rather difficult, and often impossible. There are also other issues to consider, when assessing the accuracy of the analysis. Some of them are:

1. Limited number of FEM elements used to create meshes for the vibrating system and the air. Also, finite size of the elements will results in degraded accuracy at higher frequencies.
2. The model is simplified. This was done to reduce the number of elements and subsequently the number of mesh nodes,
3. Some mechanical dimensions (bobbin height and diameter, cone) simply can not be measured without disassembling the actual driver. These dimensions were mostly second-guessed.
4. Main dimensions of the driver are measured with finite accuracy – typical 2mm.
5. Simplification of mathematics by decoupling the system, and incorporating "added mass" approach.
6. Voice coil and bobbin are lumped into one layer of elements. This was done to reduce the number of elements and subsequently the number of mesh nodes, Cone, bobbin and suspension thickness was assumed as 1mm.

Taking into account the above, it is understood, that we are looking for a broad agreement between the measured data and results of FEM modeling. It is expected, that some cone resonances may be missing altogether, some resonances will be located at somehow different frequencies, and some resonances will manifest themselves with different amplitudes. It is also clear, as to what are the possible immediate ways to improve the modeling accuracy.

3. Radiation into free-space

The operational model of working loudspeaker described above assumes, that the loudspeaker radiates into a free space. In other words, there is no boundary eclipsing the air in front of the cone. Failing to satisfy this condition would create an enclosed, resonating air-space in front of the cone, that would imprint it's own resonances on the sound generated by the vibrating cone. We would not be able to discriminate between these parasitic air resonances, and cone break-up resonances. This condition needs to be taken into account, when using FEM to model the air in front of the cone. The FEM is easily applicable to enclosed spaces, but some extra work needs to be done for open spaces.

One obvious possible solution is to introduce a special, layer of modified elements or nodes, which will act as a perfect absorber for the incoming sound waves, generated by the cone. Therefore no acoustic energy would be reflected back to the driver's cone. We require, that an plane acoustic wave incident upon an absorptive material was completely absorbed by the material. In this situation, that absorption coefficient of the material, α, needs to be equal to unity ($\alpha = 1$). Acoustic impedance, Z_a of a surface, S, can be expressed by the following relation:

$$Z_s(\omega) = (R_{ms} + j\omega M_{ms} + \frac{1}{j\omega C_{ms}})/S$$

Where Rms = losses, Mms = material mass, Cms = material suspension. Complete absorption will be accomplished, if the acoustic impedance of the absorbing surface is equal to the acoustic impedance of the air. This is a simple impedance matching:

$$Z_s = Z_{air} = \rho c \quad -> \quad \alpha = 1$$

Where c = 344m/s (speed of sound), and ρ = 1.18kg/m^3 is the air density. Accordingly to Petyt[2], boundary surface absorption condition, for enclosed air space with an acoustic pressure P within the enclosure, is expressed as:

$$\frac{\partial P}{\partial n} = -i\rho\omega \frac{P}{Z_s}$$

Where: P = pressure within enclosed space, n = vector normal to the surface, ρ = 1.18kg/m^3 is the air density, and Zs is the acoustic impedance of the surface. In FEM modelling, the above condition equates to introducing an absorbing matrix of nodes, where each element of the matrix is defined as:

$$d_e = \int_{S_e} \frac{\rho}{Z_s} N_i^T[]N_i[]ds$$

Ni are shape elemental function, N^T is a transposed Ni matrix, and depends on the type of FEM element used. Please note, that the above integral is a surface integral, calculated over the absorbing surface, Se. We are in the position to calculate this integral, as all variables are known, and Zs = ρc. Shown below, is the air mesh used in the model, with the location of the absorbing nodes marked in red.

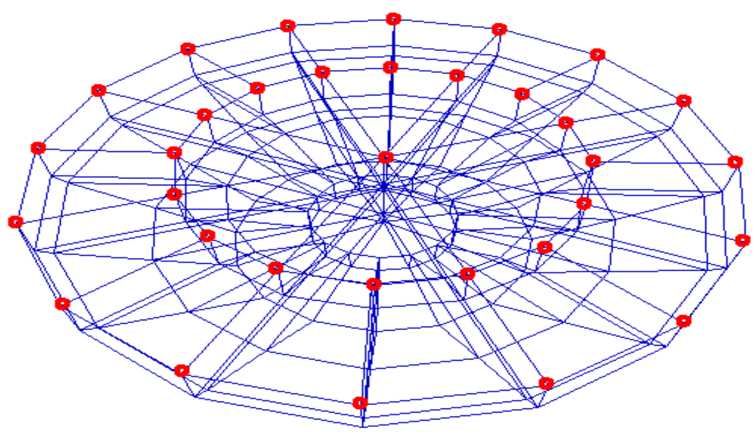


Figure 3. Mesh representing loudspeaker's air load. Absorbing nodes are marked in red.

4. FEM representation of the vibrating (structural) system

$$\{ K[][] - \omega^2 M[][] + j\omega D[][] \} U[] = -j\omega F[]$$

The above equation is solved for the nodal displacement vector, U[]. Nodal displacements are actually cone displacements, so in the next step, it will be possible to use these results to simulate the cone movement that excites the air in front of the cone, resulting in sound radiation from the system.

F[] is the excitation vector created by **F = Bli** force from the voice coil. Where BL is the force factor of the magnet assembly and "i" is the current flowing through the voice coil.

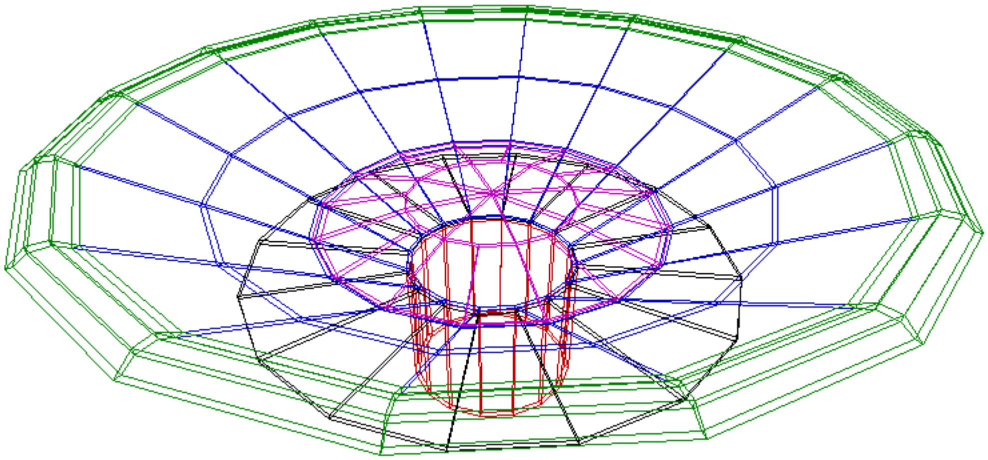
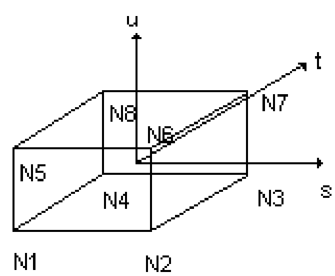


Figure 4. Mesh representing loudspeaker's vibrating components

shape functions defined Elements k[][] of the stiffness matrix K[][] can be expressed as follows:

$$k_i[][] = \int_{Vi} B_i^T[][] D[][] B_i[][] dV$$

Elemental matrix for 8-node element are 8x8 in size. Vector N[] represents eight linear as below:



$N_1 = (1 - s)(1 - t)(1 - u)/8$ $N_2 = (1 + s)(1 - t)(1 - u)/8$
$N_3 = (1 + s)(1 + t)(1 - u)/8$ $N_4 = (1 - s)(1 + t)(1 - u)/8$
$N_5 = (1 - s)(1 - t)(1 + u)/8$ $N_6 = (1 + s)(1 - t)(1+ u)/8$
$N_7 = (1 + s)(1 + t)(1 + u)/8$ $N_8 = (1 - s)(1 + t)(1 + u)/8$

Strain matrix [B] is of the form: $[B] = [B_1\ B_2\ B_3\ B_4\ B_5\ B_6\ B_7\ B_8\ ]$

$$B_i = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \end{bmatrix}$$

and, for isotropic material, the stress and strains are related by the constitutive matrix D[][]:

$$D = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) \end{bmatrix}$$

Material properties are as follows:

	E=Young's Modulus	**Density ρ**	**Poisson's ratio ν**	**Thickness**
Paper Cone	$5.832*10^9$	320.0	0.30	1mm
Rubber Surround	$6.630*10^7$	1124.0	0.48	1mm
Paper Dust Cup	$5.832*10^9$	320.0	0.30	1mm
Voice Coil+Bobbin	$1.003*10^{11}$	320.0	0.30	1mm
Cloth suspension	$8.000*10^8$	120.0	0.30	1mm

The elements m[][] of the inertia matrix M[][] are expressed as follows:

$$m_i[][] = \int_{Vi} \rho N_i^{\,T}[] N_i[] dV$$

Ni are shape elemental function, N^T is a transposed Ni matrix, and depends on the type of FEM element used, $\rho = 1.18 kg/m^3$ is the air density

5. FEM representation of the air-load in front of the cone

FEM equation describing the behavior of the air in front of the cone can be arranged as follows:

$$\{ K[][] - \omega^2 M[][] + j\omega D[][] \} P[] = -\omega^2 U[]$$

Elements k[][] of the stiffness matrix K[][] can be expressed as follows:

$$k_i[][] = \int_{Vi} B_i^{\,T}[][] B_i[][] dV$$

Where matrix B[][] is expressed as:

$$B_i = \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{bmatrix}$$

The complete B[][] matrix is shown below.

$$[B]=\begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial x} & \frac{\partial N_5}{\partial x} & \frac{\partial N_6}{\partial x} & \frac{\partial N_7}{\partial x} & \frac{\partial N_8}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y} & \frac{\partial N_4}{\partial y} & \frac{\partial N_5}{\partial y} & \frac{\partial N_6}{\partial y} & \frac{\partial N_7}{\partial y} & \frac{\partial N_8}{\partial y} \\ \frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} & \frac{\partial N_3}{\partial z} & \frac{\partial N_4}{\partial z} & \frac{\partial N_5}{\partial z} & \frac{\partial N_6}{\partial z} & \frac{\partial N_7}{\partial z} & \frac{\partial N_8}{\partial z} \end{bmatrix}=$$

$$\begin{bmatrix} J_{00} & J_{01} & J_{02} \\ J_{10} & J_{11} & J_{12} \\ J_{20} & J_{21} & J_{22} \end{bmatrix}^{-1}\begin{bmatrix} \frac{\partial N_1}{\partial s} & \frac{\partial N_2}{\partial s} & \frac{\partial N_3}{\partial s} & \frac{\partial N_4}{\partial s} & \frac{\partial N_5}{\partial s} & \frac{\partial N_6}{\partial s} & \frac{\partial N_7}{\partial s} & \frac{\partial N_8}{\partial s} \\ \frac{\partial N_1}{\partial t} & \frac{\partial N_2}{\partial t} & \frac{\partial N_3}{\partial t} & \frac{\partial N_4}{\partial t} & \frac{\partial N_5}{\partial t} & \frac{\partial N_6}{\partial t} & \frac{\partial N_7}{\partial t} & \frac{\partial N_8}{\partial t} \\ \frac{\partial N_1}{\partial u} & \frac{\partial N_2}{\partial u} & \frac{\partial N_3}{\partial u} & \frac{\partial N_4}{\partial u} & \frac{\partial N_5}{\partial u} & \frac{\partial N_6}{\partial u} & \frac{\partial N_7}{\partial u} & \frac{\partial N_8}{\partial u} \end{bmatrix}$$

Next, the elements m[][] of the inertia matrix M[][] are expressed as follows:

$$m_i[][]=\int_{Vi}\rho N_i^T[]N_i[]dV$$

Both elemental matrixes for 8-node element are 8x8 in size. Vector N[] represents eight linear shape functions defined as for the structural part.

Finally, the elements d[][] of the absorption matrix D[][] are expressed as:

$$d_i[][]=\frac{1}{c}\int_{Si}N_i^T[]N_i[]dS$$

Elements d[][] are surface elements, located at the common surface of cone-air interface. This is why the integral is a surface integral rather then volume integral.

Due to complexity of the resulting expressions for element k[][] and m[][] they are best evaluated by using numerical integration techniques. Integration can be facilitated by Gauss integration scheme, requiring integration limits of +1 and –1. We can change integration from x,y,z space to s,t,u space with the help of Jacobian J[][] of the transformation resulting immediately in the volume element dV being expressed as follows:

$$dV=|J|dsdtdu$$

Where |J| is the determinant of Jacobian matrix J[][].

Jacobian of the Transformation

$$\begin{bmatrix} \frac{\partial N_i}{\partial s} \\ \frac{\partial N_i}{\partial t} \\ \frac{\partial N_i}{\partial u} \end{bmatrix}=[J]\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{bmatrix}$$

Where J[] is a ***Jacobian*** of the transformation expressed as follows:

$$[J]=\begin{bmatrix}\frac{\partial N_1}{\partial s} & \frac{\partial N_2}{\partial s} & \frac{\partial N_3}{\partial s} & \frac{\partial N_4}{\partial s} & \frac{\partial N_5}{\partial s} & \frac{\partial N_6}{\partial s} & \frac{\partial N_7}{\partial s} & \frac{\partial N_8}{\partial s} \\ \frac{\partial N_1}{\partial t} & \frac{\partial N_2}{\partial t} & \frac{\partial N_3}{\partial t} & \frac{\partial N_4}{\partial t} & \frac{\partial N_5}{\partial t} & \frac{\partial N_6}{\partial t} & \frac{\partial N_7}{\partial t} & \frac{\partial N_8}{\partial t} \\ \frac{\partial N_1}{\partial u} & \frac{\partial N_2}{\partial u} & \frac{\partial N_3}{\partial u} & \frac{\partial N_4}{\partial u} & \frac{\partial N_5}{\partial u} & \frac{\partial N_6}{\partial u} & \frac{\partial N_7}{\partial u} & \frac{\partial N_8}{\partial u}\end{bmatrix}\begin{bmatrix}x1 & y1 & z1 \\ x2 & y2 & z2 \\ x3 & y3 & z3 \\ x4 & y4 & z4 \\ x5 & y5 & z5 \\ x6 & y6 & z6 \\ x7 & y7 & z7 \\ x8 & y8 & z8\end{bmatrix}=$$

$$\begin{bmatrix}\sum_{i=1}^{8}\frac{\partial N_i}{\partial s}x_i & \sum_{i=1}^{8}\frac{\partial N_i}{\partial s}y_i & \sum_{i=1}^{8}\frac{\partial N_i}{\partial s}z_i \\ \sum_{i=1}^{8}\frac{\partial N_i}{\partial t}x_i & \sum_{i=1}^{8}\frac{\partial N_i}{\partial t}y_i & \sum_{i=1}^{8}\frac{\partial N_i}{\partial t}z_i \\ \sum_{i=1}^{8}\frac{\partial N_i}{\partial u}x_i & \sum_{i=1}^{8}\frac{\partial N_i}{\partial u}y_i & \sum_{i=1}^{8}\frac{\partial N_i}{\partial u}z_i\end{bmatrix}$$

Now, the derivatives of the shape functions in respect to x,y and z coordinates can be calculated by inverting the Jacobian matrix J[][].

$$\begin{bmatrix}\frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z}\end{bmatrix}=[J]^{-1}\begin{bmatrix}\frac{\partial N_i}{\partial s} \\ \frac{\partial N_i}{\partial t} \\ \frac{\partial N_i}{\partial u}\end{bmatrix}$$

6. Coupling the FEM Model to the Amplifier

Loudspeaker's cone is moved by applying electrical voltage, **V**, across the terminals of voice coil. This voltage will create current ,**i**, flawing through the voice coil. The total electrical resistance Ze, seen by the amplifier can be expressed as:

$$Ze = Re + j\omega Le, \quad \text{and} \quad i=\frac{V}{Ze}$$

Where Re = DC resistance of the voice coil, Le = electrical inductance of the voice coil, and ω = 2πf, f = frequency of the applied signal. The Ze impedance excludes motional impedance, and can also be measured by blocking movements of the voice coil. We can now couple the force applied to the cone to the output voltage, **V**, from the amplifier as:

$$F = BLi = \frac{BLV}{Ze} = \frac{BLV}{R_e + j\omega L_e}$$

Where BL, is the force factor from driver's specifications.

In summary:

Amplifier voltage, V, will create force, F, as per following formula:

$$F = BLi = \frac{BLV}{Ze} = \frac{BLV}{R_e + j\omega L_e}$$

Force F, applied to the voice coil will result in cone displacements, U, as per formula below:

$$\{ K[][] - \omega^2 M[][] + j\omega D[][] \} U[] = -j\omega F[]$$

Cone displacements, U will eventually create sound pressure P, as per formula below:

$$\{ K[][] - \omega^2 M[][] + j\omega D[][] \} P[] = -\omega^2 U[]$$

As a result, we will obtain a series of N nodal pressures, Pi:

$$Pi(j\omega) = Re\{Pi\} + j\,Im\{Pi\}, \qquad i = 0\ldots\ldots\ldots N$$

7. Combining pressure from N-radiating nodes

Sound pressure calculated so far at all radiating nodes needs to be combined at the measurement point.

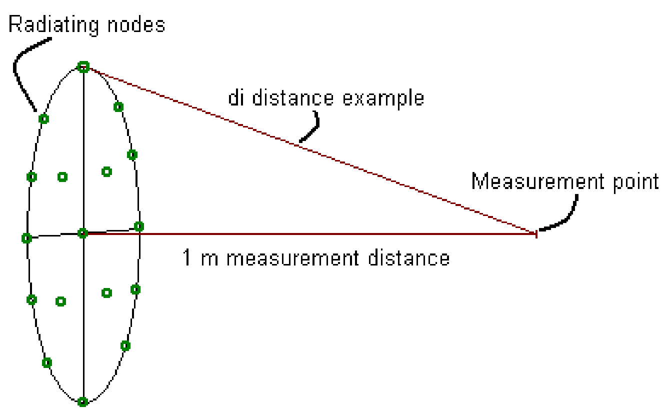


Figure 5. Example of di distance calculations.

To accomplish this, individual nodal pressures, Pi, are simply added together, accounting for a different delays due to different distances between each radiating node and the summing point. The summing (measurement) point is assumed to be 1meter from the radiating plane of the air mesh.

Pressures, Pi, from each node are combined as follows:

$$P(j\omega) = \Sigma_i \left[(Re\{Pi\} + j\,Im\{Pi\})\, e^{(j\,\omega\, di / c)}\right] =$$
$$\Sigma_i \left[(Re\{Pi\} + j\,Im\{Pi\})\,(\cos(\omega\, d_i / c) - j \sin(\omega\, d_i / c))\right], \quad i = 0\ldots\ldots N$$

Where: di = is the distance from each node to the summing point, c = 344m/s (speed of sound), and $\omega = 2\pi f$, f = frequency of the applied signal. The exponent in the above expression represents the arrival time delay due to the distance di.

8. Calculating Final SPL Curve

The final SPL curve modeling includes the following components:

1. Standard sealed enclosure transfer function.
2. Diffraction distortion dues to enclosure edges.
3. Influence of the voice coil inductance Le.
4. Transfer function of the cone break-up effects.

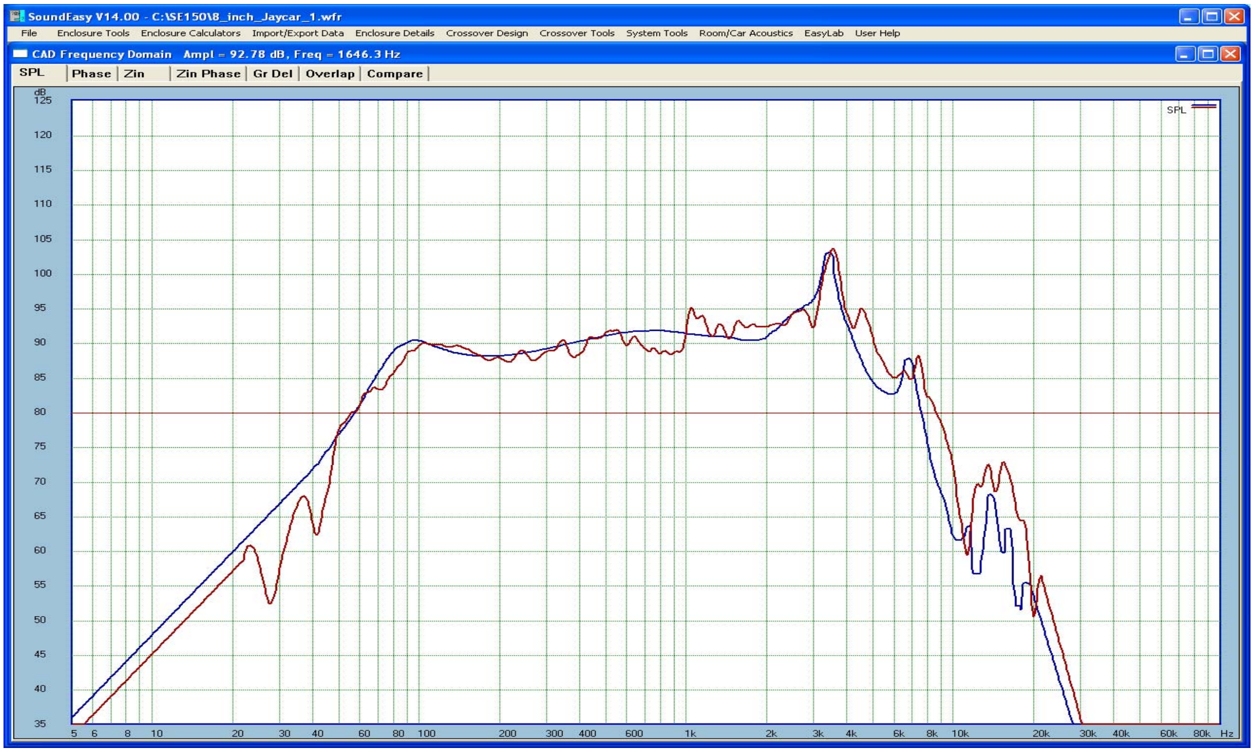


Figure 6. Comparison of Calculated (BLUE) and measured (RED) SPL curves of a small woofer.

As shown on Figure 6, a good agreements between measured (at 1 meter) and calculated SPL curves was achieved, particularly, when taking into account limitations of the model listed in Section 2. Most of the cone break-up resonances were re-created by the model at correct frequencies and with acceptable amplitudes. Also, the overall frequency response roll-off towards higher frequencies is in agreement with the measured results. The loudspeaker was measured in-room, so the quality of the SPL curve is clearly affected by the environment.

The double-FEM model has been explained, and coupling of the model to the amplifier on one side, and to the air on the other side was given.

It is important to realize, that the blue curve on Figure 6 is entirely due to the mathematical model including the 4 components listed above. This is a very significant progress from the days of loudspeaker modeling, when things were based exclusively on T/S parameters. Indeed, current loudspeaker modeling programs are capable of:

1. Modeling enclosure SPL
2. Modeling port resonances and compression effects.
3. Modeling thermal effects due to the voice coil
4. Modeling non-linear effects of BL and vibrating system
5. Modeling enclosure resonances
6. Modeling diffraction distortions
7. And finally - modeling cone break-up effects

Capability (7) has been implemented using Finite Element Method. As typical with FEM, there is a tread-off between speed of the method and accuracy at higher frequencies, as both depend on the number of nodes included in the model.

9. Conclusions

A simplified FEM system was developed to model cone break-up effects on electro-dynamic transducer's frequency response. Several assumptions were taken into consideration, as explained in Section 2. The results of the modeling, are in broad agreement with measured frequency response of a small woofer tested – see Figure 6.

Usefulness of the model is contingent upon availability of mechanical properties of the materials used to manufacture the loudspeaker, and also on detailed mechanical dimensions of all moving parts of the loudspeaker assembly. Unfortunately, these requirements can be difficult to satisfy for any driver. Mechanical properties of the materials are not widely published, as they are often a proprietary information, making one driver sounding better than the other. As a substitute solution, a method could be developed to allow for isolation of individual elements of the vibrating system and then tune some default materials parameters' , using the measured frequency response as a template. However, this almost defeats the purpose of explicit modeling itself.

If all of the required parameters are known, it is very useful to incorporate extended frequency response in the design process. For once, crossover cut-off points can be determined more accurately, and also, the summed frequency response is more accurate and realistic – just to mention a few advantages.

Finally, if all the required parameters are known, it seems that more accurate way of obtaining the broad-band frequency response of a loudspeaker would be to measure the SPL using close-mike technique (up to 200Hz, in our example), then add diffraction effects (up to 1.5kHz, in our example), and finally merge into it the cone break-up effects (above 1.5kHz in our example). The resulting frequency response, as seen on Figure 6, would be useful for modeling purposes to above 10kHz for this woofer.

References:

1. P. Bettess (Ed.), 1978, `Fluid-structure interaction', Special edition of Int. Journ. Num. Meth. Engng., 13(1).
2. M. Petyt – Finite Element for Acoustics.