Inverse Hilbert_Bode Transform

By Bohdan Raczynski, Bodzio Software Pty. Ltd., Feb 2019

1. Introduction and remainder on HBT

Before we introduce Inverse Hilbert-Bode Transform (IHBT), a short summary of the original Hilbert-Bode Transform (HBT) is presented.

The HBT was introduced to the DIY community about 20 years ago and it is a computational algorithm, that allows the user to extract phase response from the known magnitude response, or SPL curve. Since the HBT is based on an integral calculated from DC to infinity, one can immediately see the problem of supplying SPL data points over this impossible frequency range. However, it turns out, that calculating the integral over a narrower and much more manageable frequency range affects the total phase accuracy only in a minimal way. However, the other problem of extending the asymptotic slopes of the SPL curve towards zero frequency and infinite frequency still remains.

Some substitute methods for determining the slopes have been proposed, and the method of "Optimized Guiding Filter" is possibly the most accurate approximation. The IHBT properties make it perhaps more suitable for solving the above dilemma.

2. Introduction to "Phase Slopes" Corresponding to SPL Slopes.

When dealing with traditional HBT the user needs to supply asymptotic SPL slopes, whose tangent is expressed in dB/oct. The SPL slopes, together with the measured SPL constitute input data into the HBT algorithm. We are all familiar with SPL slopes.

Similarly, the IHBT requires **asymptotic phase slopes** to be attached to the phase response at selected frequency points. This combined phase response constitutes input data into the IHBT algorithm. Here is the problem: we are not really familiar with the concept of "phase slopes" expressed in deg/oct.

In order to make the IHBT a bit more digestible, the phase slopes (or tails) are calculated from the equivalent SPL slopes. In the IHBT "language" this will be called "High-Pass PHASE Tail Equivalent To SPL" slope attached at say, 35Hz with 24dB/oct slope. From this data, the algorithm will calculate corresponding phase slope and will attach it to the measured phase at the specified frequency point. Same deal for "Low-Pass PHASE Tail Equivalent To SPL" slope.

3. SPL from Phase for Simple Filters

In order to start visualizing the operation of IHBT, a couple of simple filters were constructed and their phase responses were supplied to IHBT algorithm.

Figure 1 below, shows a simple band-pass filter with +/-24dB/oct slopes and a shelving component. The thick black curve, which is the SPL calculated via IHBT, actually overlaps the original SPL curve, so it can not be seen.

Figure 2 below, shows a simple band-pass filter with +/-48dB/oct slopes, a shelving component and a Q_parametric component. The thick black curve, which is the SPL calculated via IHBT, actually overlaps the original SPL curve, so it can not be seen again.

Figure 1. SPL extracted from phase response of a simple filter 1.

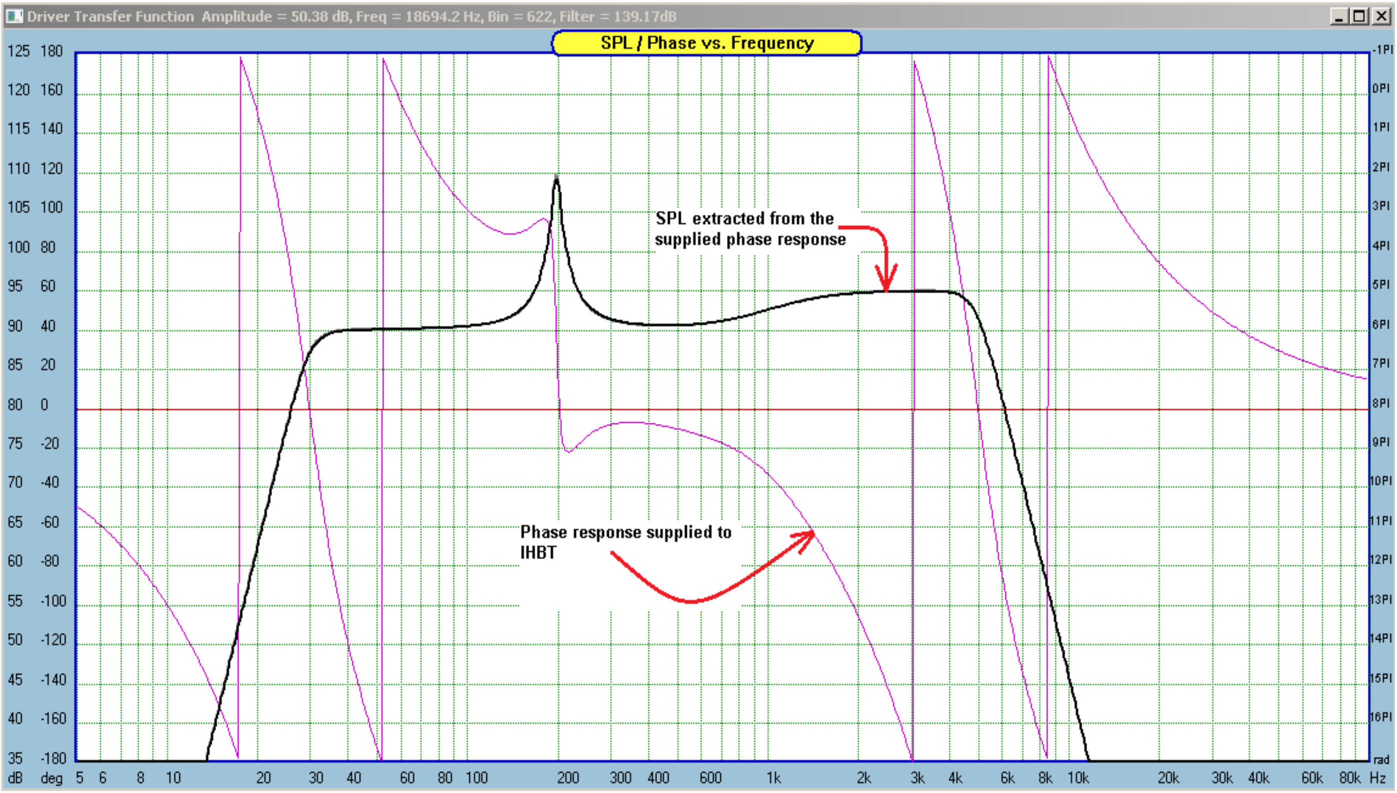


Figure 2. SPL extracted from phase response of a more complex filter 2.

Next, we will examine a real loudspeaker phase response.

4. SPL from Phase for Measured Example Loudspeaker

The MLS system was used to measure a 12” loudspeaker in vented enclosure, so the asymptotic slope on the low-frequency side would be 24dB/oct.

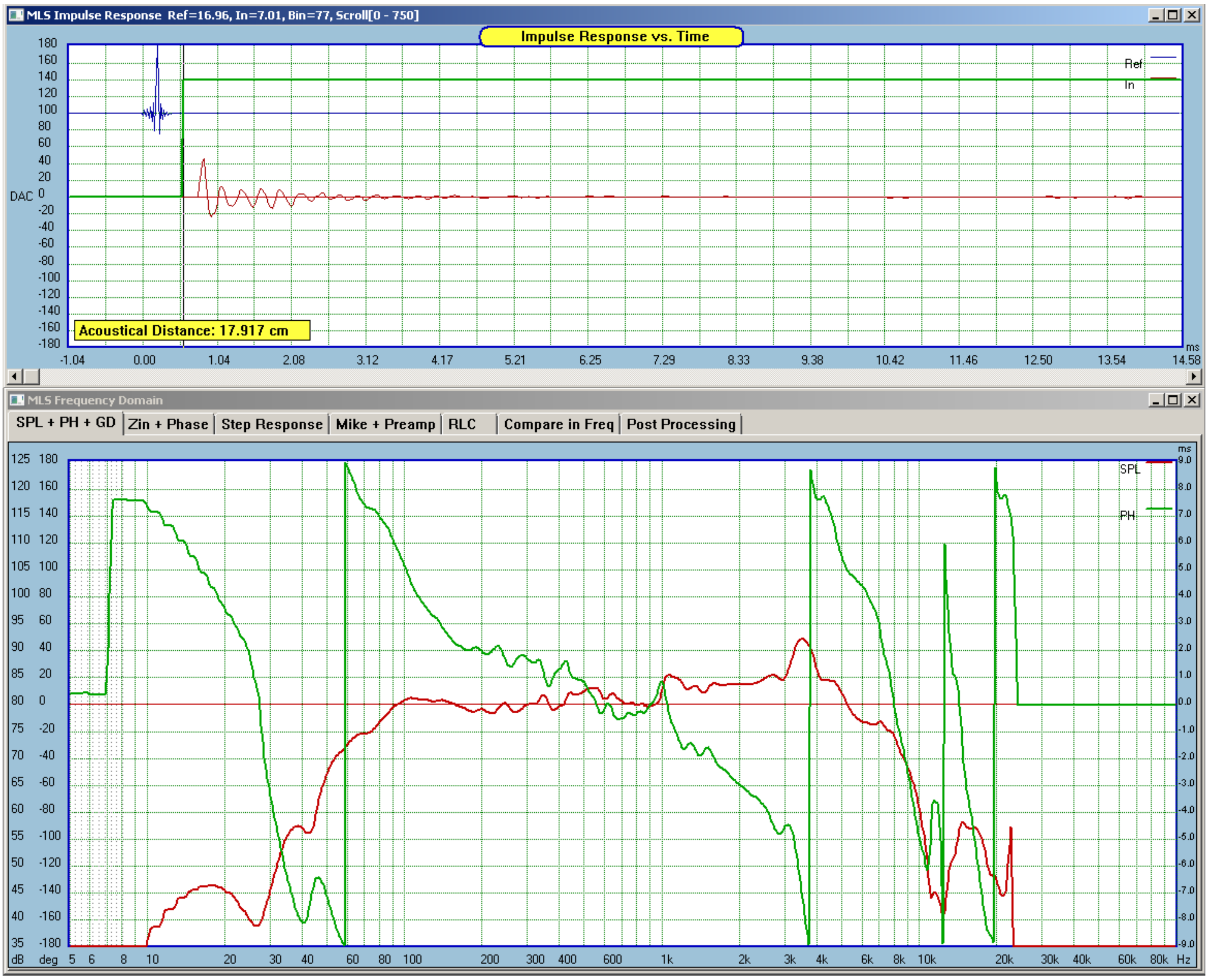
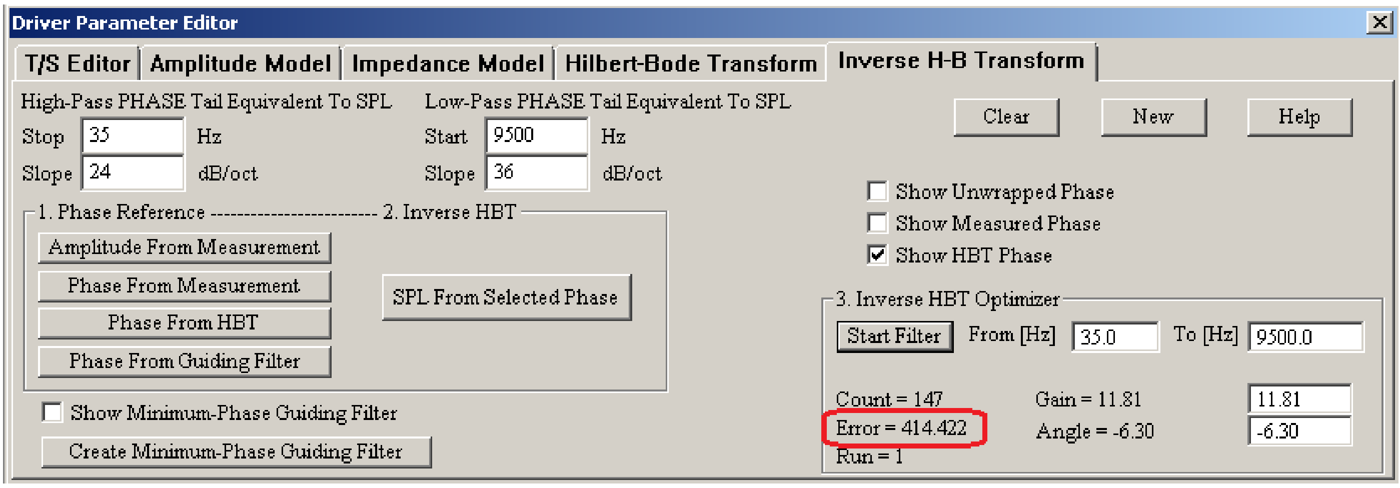


Figure 3. MLS measurement of SPL/Phase of a 12” loudspeaker with FFT window at Bin 77.

The above measured phase response (green), with FFT window set to start at Bin 77 was supplied to the IHBT algorithm. Settings for the IHBT algorithm were as follows: Stop = 35Hz, Slope = 24dB/oct and Start = 9500Hz, Slope = 36dB/oct, frequency range of IHBT optimization 35 – 9500, Start IHBT Optimizer with: Gain = 0.0, Angle = 0.0 After the run, **Error = 414**. It is observable, that SPL does not match the measured SPL from 35Hz - 3kHz, then the match is a lot better.



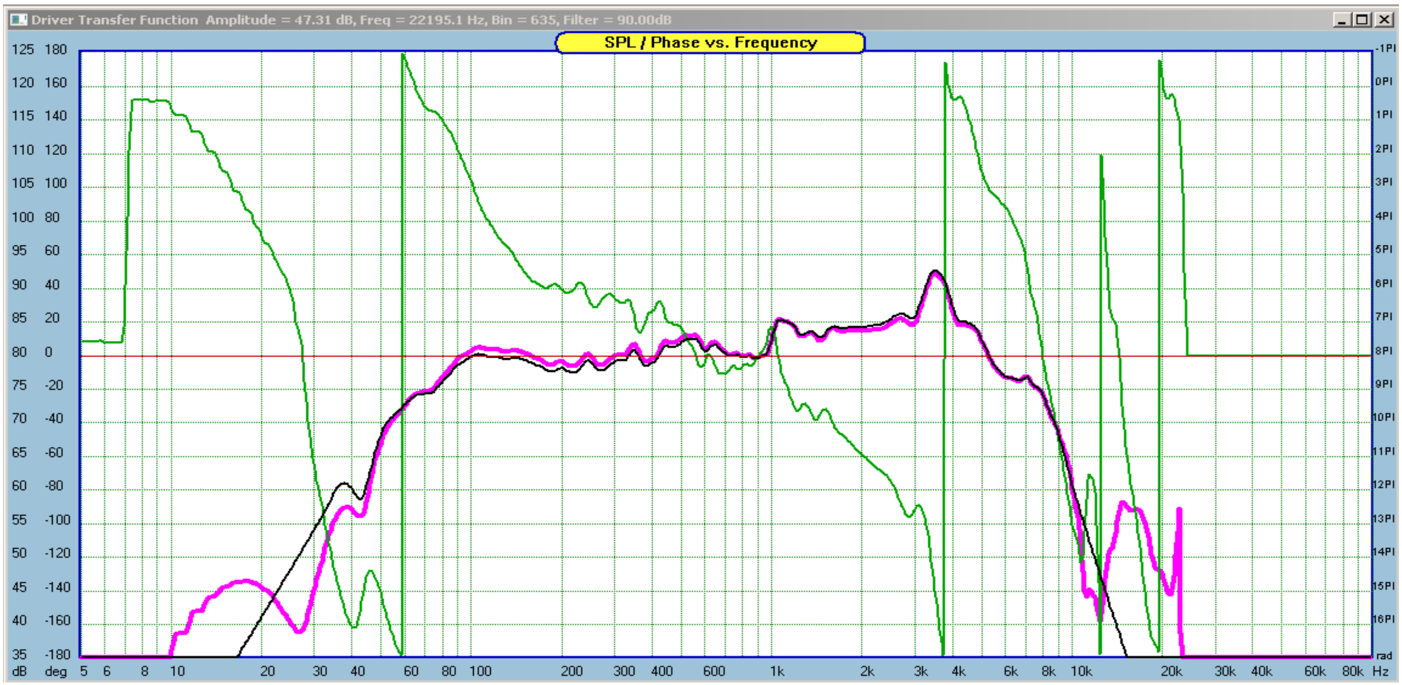


Figure 4. MLS measurement of SPL/Phase of a 12” loudspeaker with FFT window at Bin 77.

Then FFT window in MLS system was moved to Bin 76, and the process was repeated. The **Error = 511**, and was higher than for Bin 77.

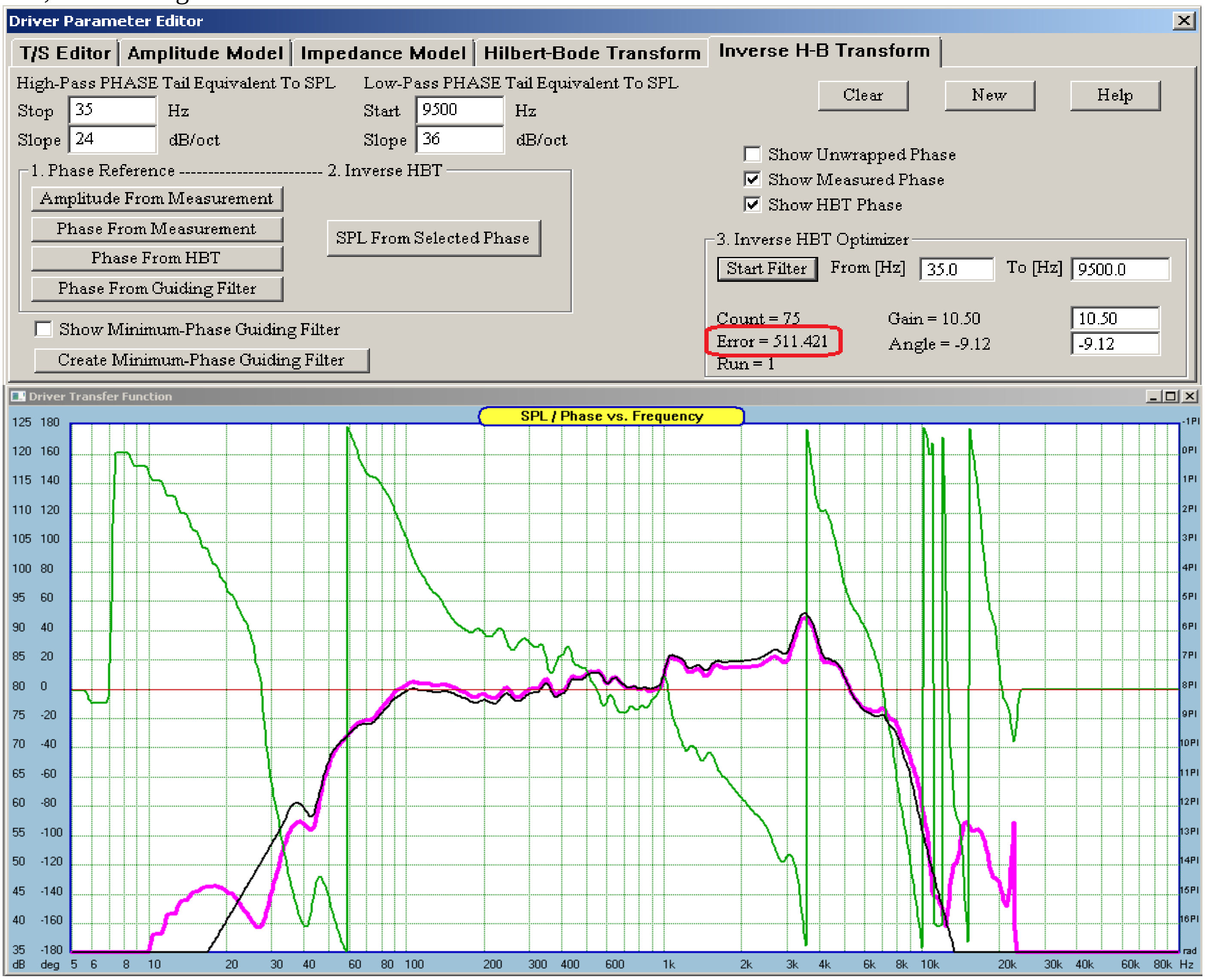
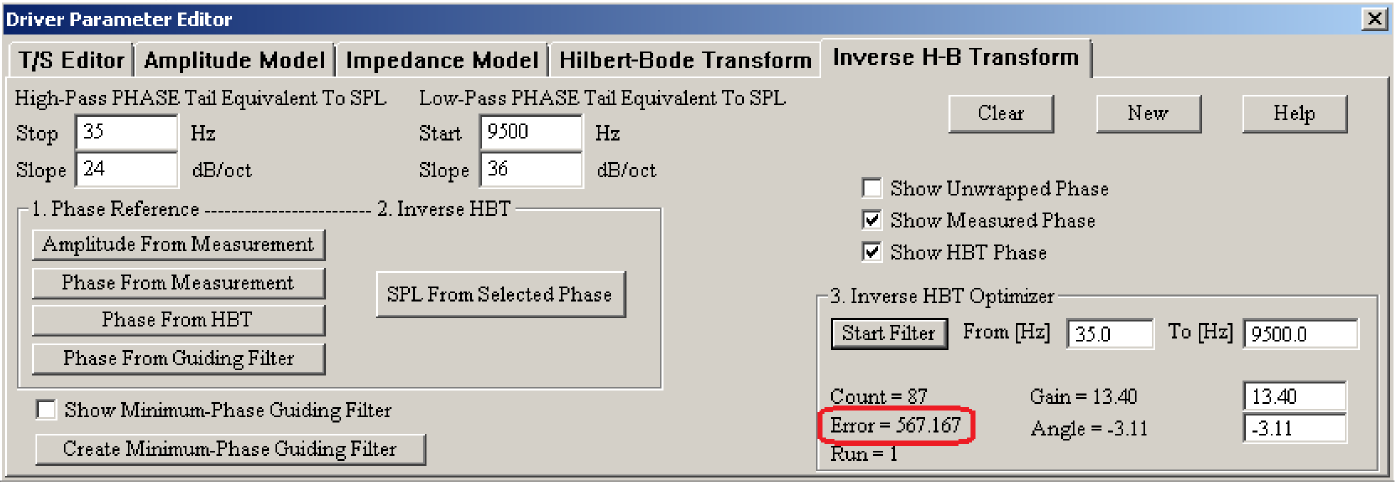


Figure 5. MLS measurement of SPL/Phase of a 12” loudspeaker with FFT window at Bin 76.

Finally, the FFT window was moved to Bin 78, and the process was repeated. The **Error = 567**, and was again higher than for Bin 77.



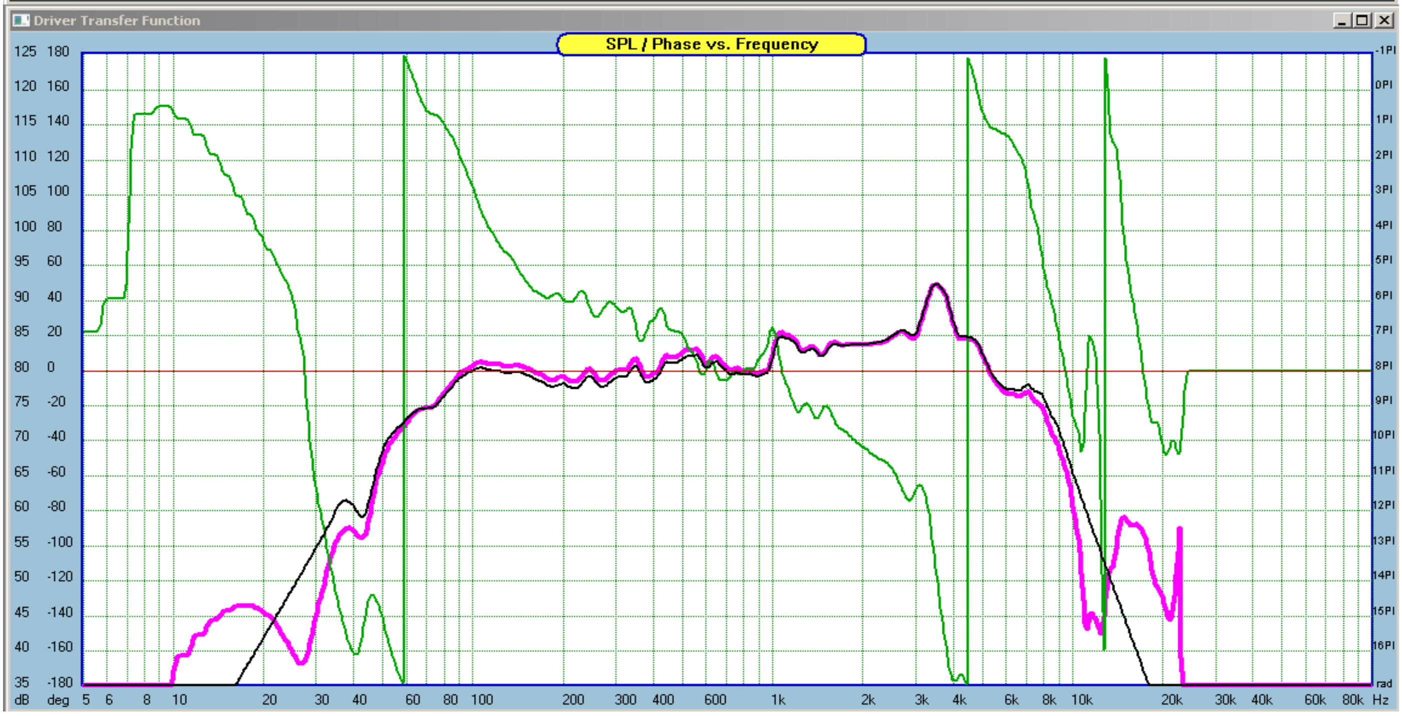
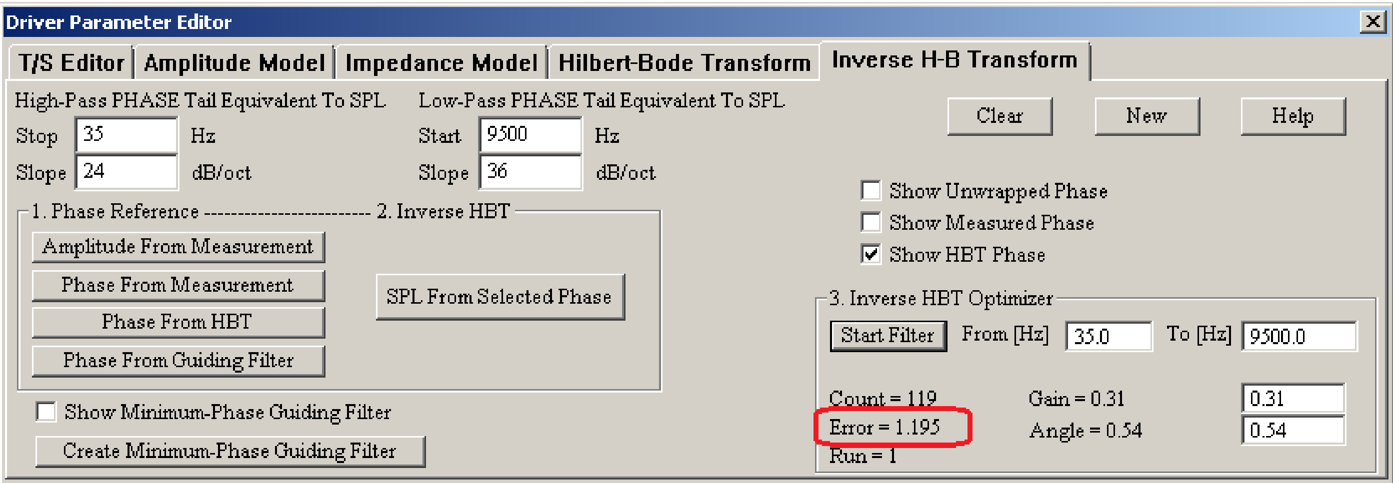


Figure 6. MLS measurement of SPL/Phase of a 12" loudspeaker with FFT window at Bin 78.

5. SPL from HBT Phase

However, when the measured SPL was used to calculate minimum-phase phase response, and this phase response was subsequently supplied to IHBT algorithm, the Error = 1.195 (very small) SPL does match the measured SPL from 35Hz – 9.5kHz



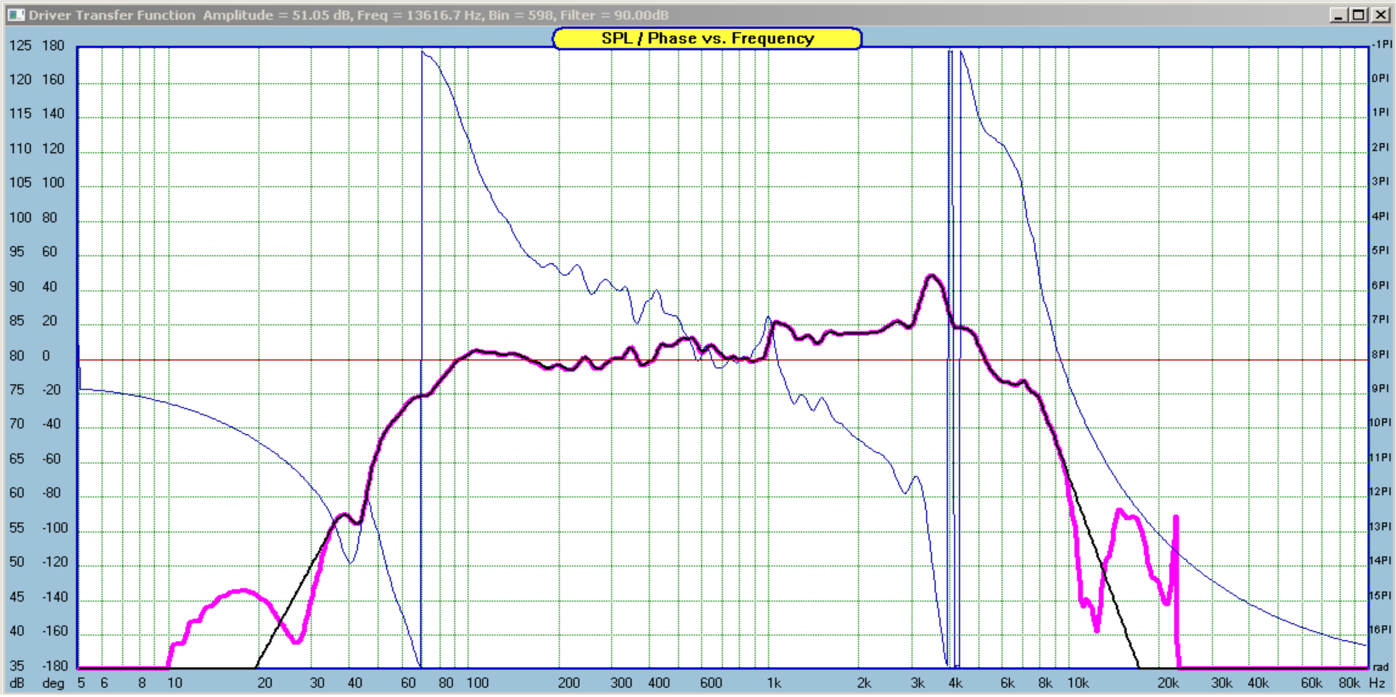


Figure 7. SPL of a 12” loudspeaker with Phase provided by HBT.

The Error is now 346 times smaller when the SPL is recovered from guaranteed minimum-phase response.

6. SPL sensitivity to attached phase slopes.

On minimum-phase systems, the accuracy of recovered SPL response can be tested by generating various phase responses and supplying such phase responses to the IHBT algorithm. In order to assure that all phase responses are indeed of the minimum-phase type, firstly, the phase response was generated by the HBT process. Then, resulting phase slopes corresponding to -18dB/oct and then -68dB/oct were attached at 9500Hz. Next, those two phase responses were supplied to the IHBT algorithm. It would be expected, that SPL would only change beyond the attachment point, as the two different phase slopes would guide the IHBT algorithm to produce corresponding SPL slopes.

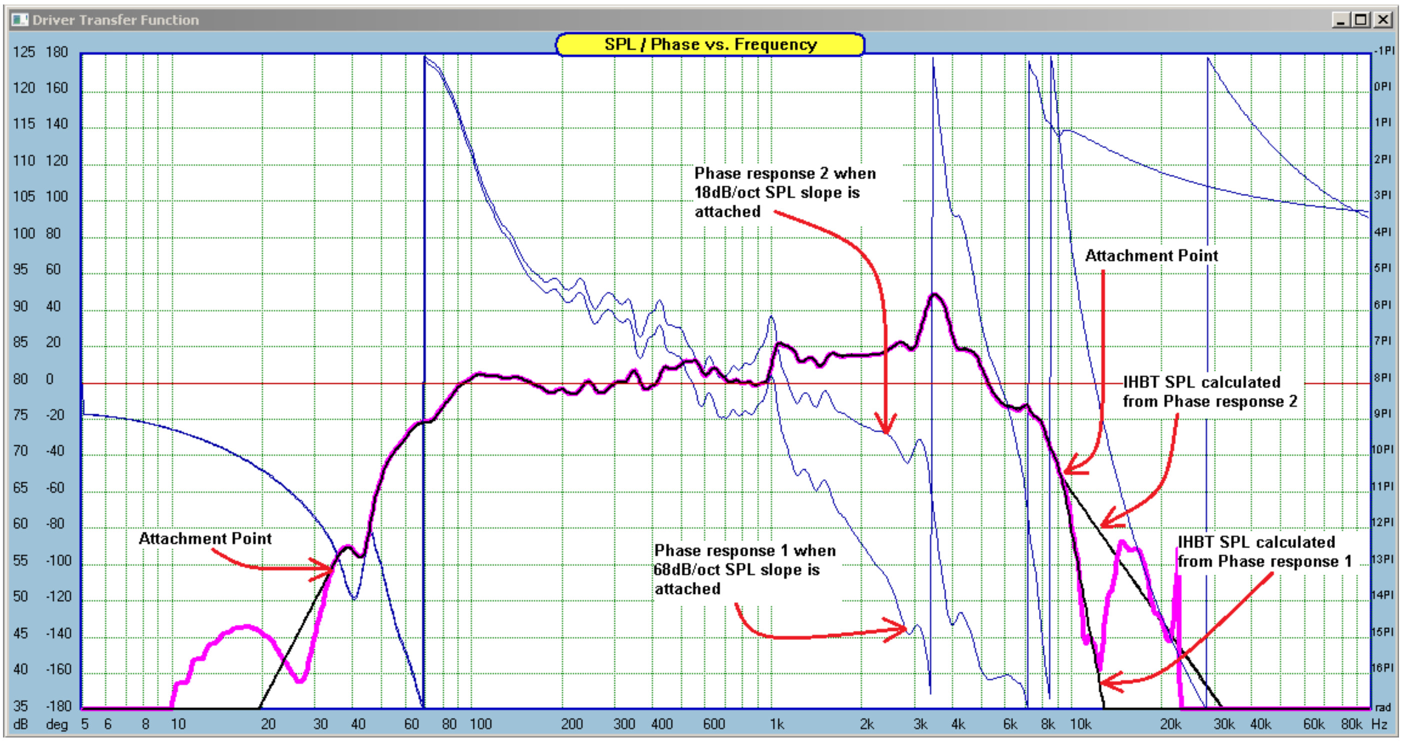


Fig 8. SPL extracted from HBT phase with attached slopes of -18dB/oct and 68dB/oct.

Indeed, for the example above, the LP slope attachment point is 9500Hz. For slopes of -18dB/oct to -68dB/oct – **NO change in extracted SPL between attachment points can be observed, even though the phase responses were dramatically different**. Similar SPL recovery accuracy can be shown for variety of SPL slopes attached at low-frequency attachment point of 35Hz. This is very important observation, as it shows, that we can use the degree of SPL match between the attachment points as the "detector" of minimum-phase characteristics, with complete disregard of the attached phase slopes. In other words, we can get the slopes wrong, but as long as the supplied phase is of the minimum-phase type, the SPL extracted between the attachment points will be perfect. Frequency range of the IHBT curve fitting mechanism should be selected to be the same as the phase slopes attachment points. As shown above, if the supplied phase response is of minimum-phase type, then the SPL will be always extracted accurately between the phase attachment points, and across the whole frequency spectrum, including the slopes. However, for the purpose of using the IHBT as a "minimum-phase detector", we are interested in evaluating the degree of SPL matching (the Error value) between the attachment points. The curve fitting algorithm manipulates two additional parameters: Gain and Angle. It is not possible to uniquely define the amplitude response from the phase data, since an infinite number of amplitude characteristics, differing only be a fixed number of dB, have identical phase response. This is where the "Gain" parameter comes in. The "Angle" parameter is related to the way mathematical functions (like arctan(x)) are calculated on the computer.

7. SPL Sensitivity to Non-minimum-phase Phase Response

This a critical question for the IHBT algorithm. How well can the IHBT discriminate between minimum-phase and non-minimum-phase data.

Measurement procedure for the example woofer.

1. Measure SPL/Phase and set FFT window to 77 bin
2. On IHBT tab and select Stop = 35Hz, Slope = 24dB/oct and Start = 9500Hz, Slope = 36dB/oct
3. Select frequency range of IHBT optimization 200 – 9500
4. Press "Phase From Measurement"-> "SPL From Selected Phase".
5. Start IHBT Optimizer with: Gain = 0.0, Angle = 0.0, **The Error = 32.579**
6. Go to MLS tab and replot SPL/Phase for 76bin
7. Go back to IHBT and press "Phase From Measurement"-> "SPL From Selected Phase".
8. Start IHBT Optimizer. **The Error = 216.77**
9. Go to MLS and replot SPL/Phase for 78bin
10. Go back to IHBT and press "Phase From Measurement"-> "SPL From Selected Phase".
11. Start IHBT Optimizer. **The Error = 35.225**

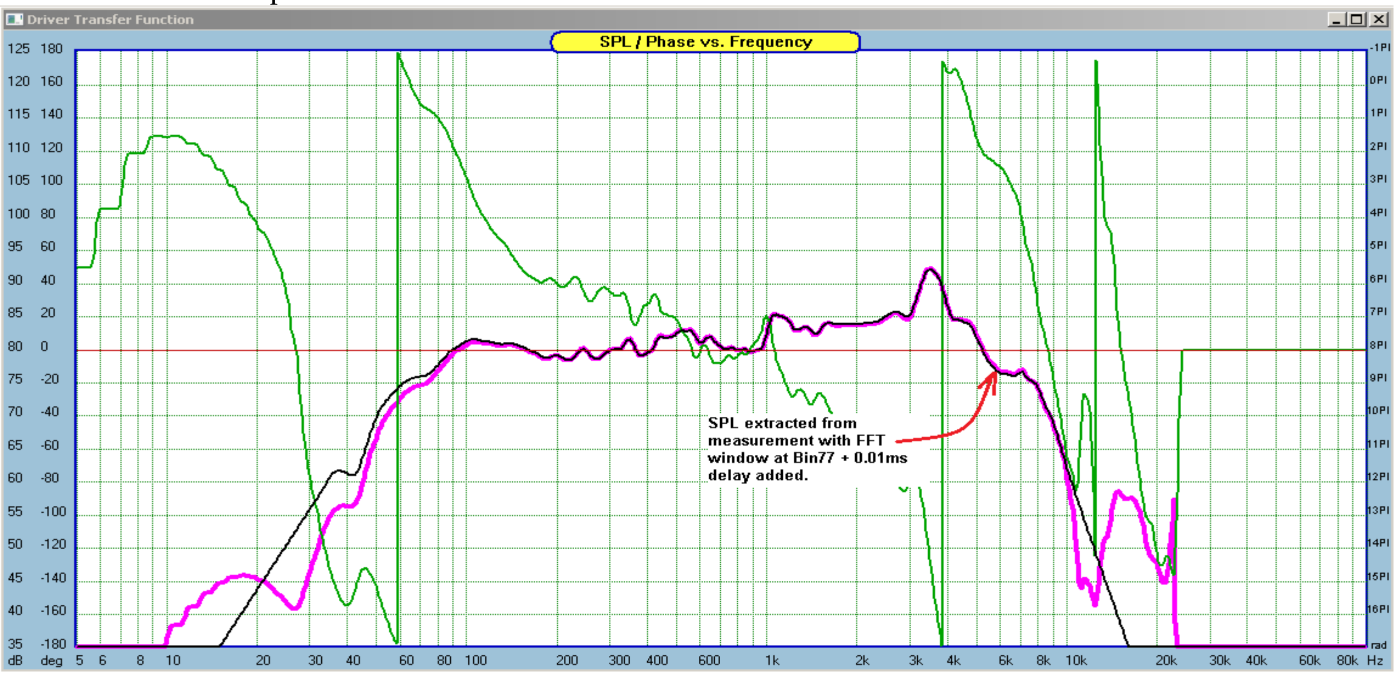
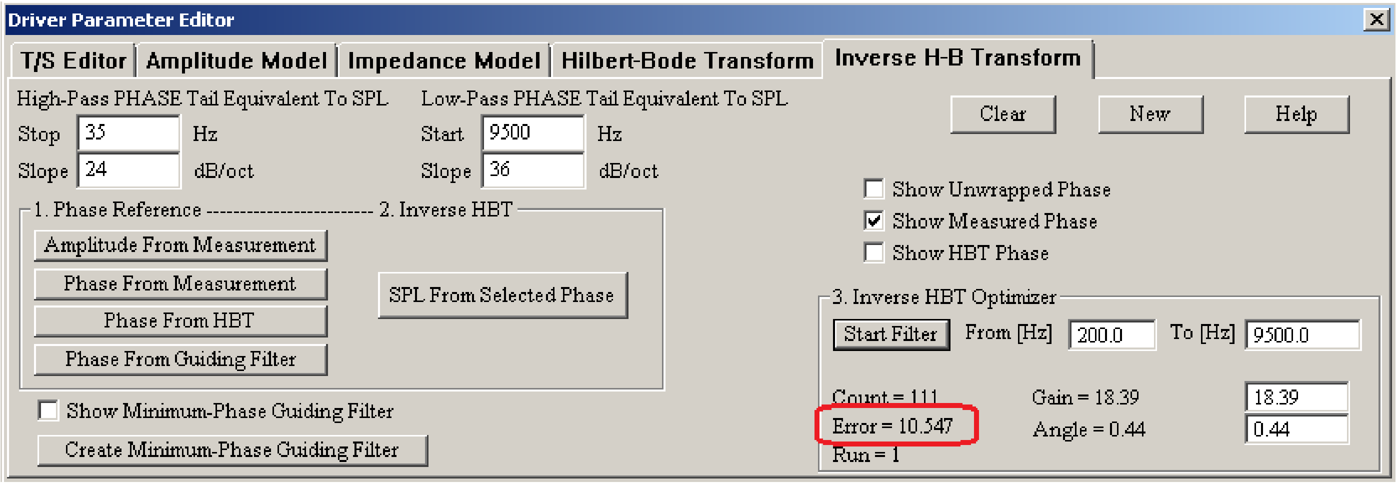


Fig 9. SPL extracted from measured phase with FFT window at Bin77+ 0.01ms delay.

Using FFT window at Bin 77, which already produced the lowest Error = 32.579, one can attempt to further reduce the error by going back to MLS system and adding small time delays. Subsequent trial errors were as follows:

Bin 78 -> Error = 35.225
Bin 77 -> Error = 32.579
Bin 77 + 0.008ms Error = 11.376
Bin 77 + 0.01ms Error = 10.547
Bin 77 + 0.012ms Error = 12.445
Bin 77 + 0.015ms Error = 15.969
Bin 76 -> Error = 216.777



In the example above, the effort was concentrated on the high-frequency range of the SPL/Phase curves. Therefore the optimization frequency range was set to 200-9500Hz. The IHBT parameters for phase slopes were set as for a vented box for the low-end, and it was demonstrated in (6), that attaching various phase slopes at high frequency end, does not affect the SPL response below the attachment point. Therefore an arbitrary slope of 36dB/oct was used for the high-end. With the above parameters, it was demonstrated, that IHBT is NOT sensitive to attached phase slopes, and will produce the same SPL curve between attachment points for a wide range of various phase slopes selected at the attachment points.

It was also demonstrated, that IHBT algorithm is quite sensitive to non minimum-phase distortions and will easily detect phase errors introduced by single FFT bin shift. Even more, in the example above, the IHBT combined with curve fitting algorithm easily detected of non-minimum-phase 1-2usc delay, which is 10 times better than single bin shift of 20.833usec at 48kHz sampling.

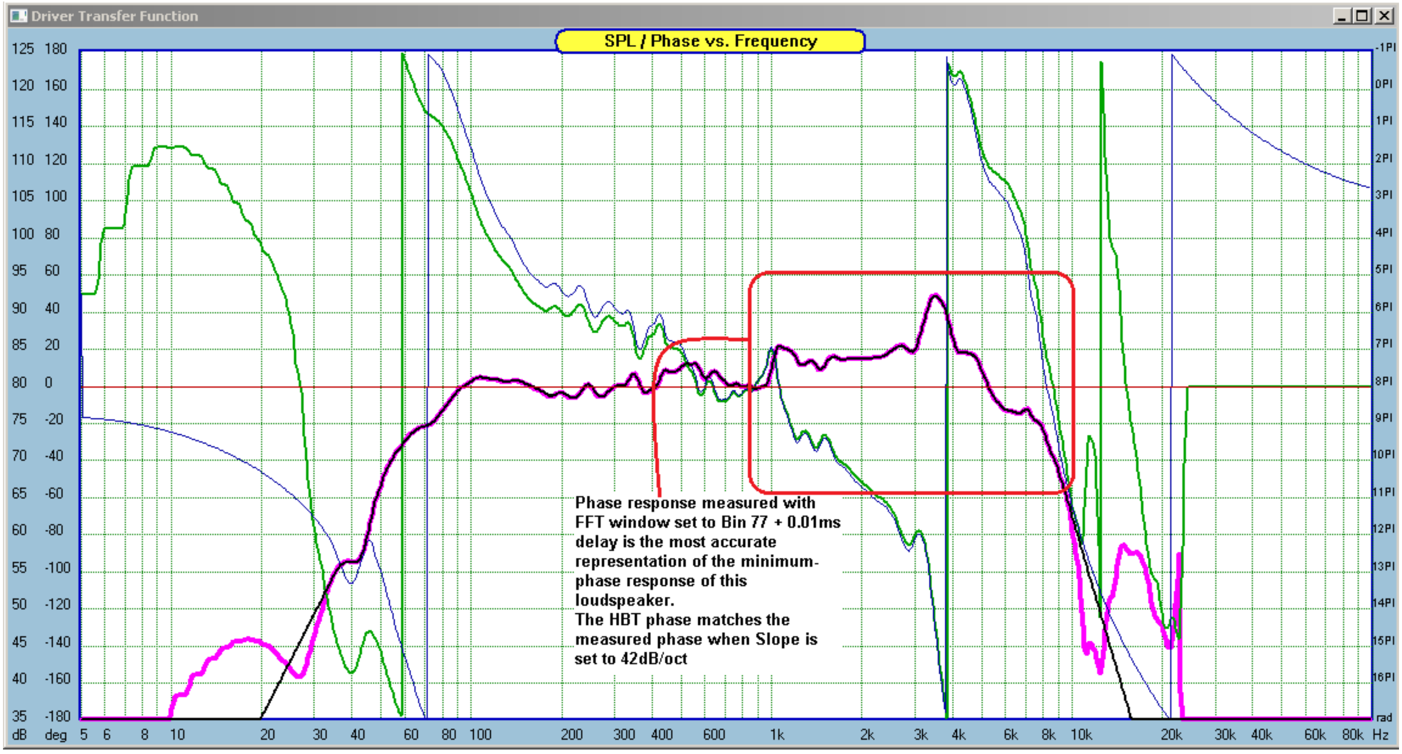
How does all of the above help with determining the minimum-phase phase response of the loudspeaker?.

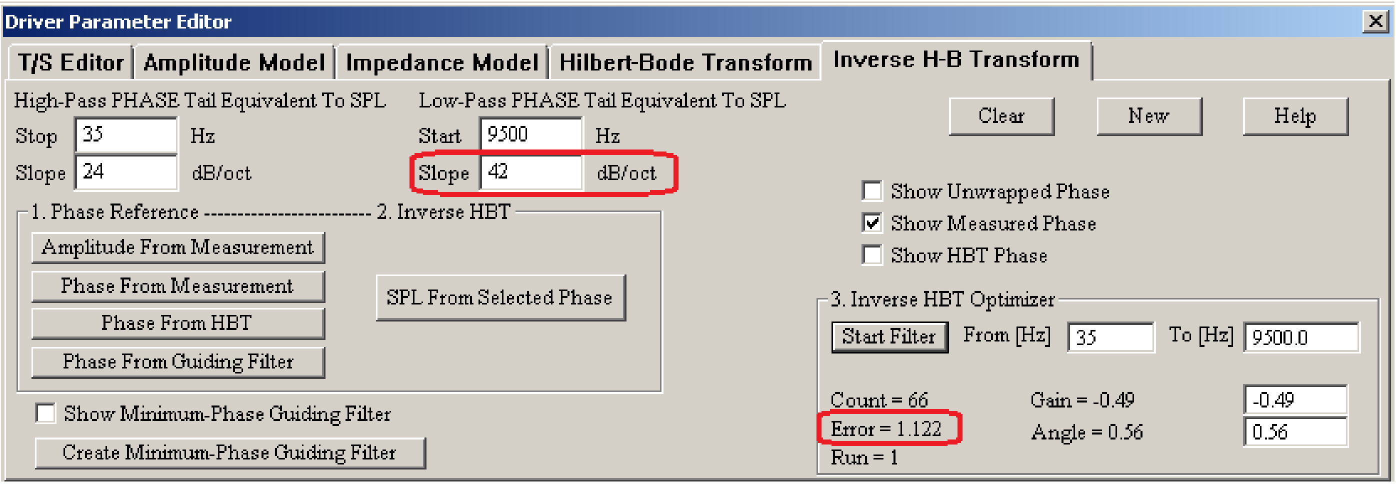
It was demonstrated, that using IHBT with the final measured phase response and FFT window set to Bin 77+0.01ms delay generates the most accurate SPL above 1kHz. This would be indicative, that supplied phase response is of minimum-phase type above 1kHz, and we have removed the non-minimum-phase component (time-of-flight). All we need to do now, is to select HBT slope that matches the final measured phase up to the attachment point of 9500Hz. Turns out, that changing the Slope from 36dB/oct to 42dB/oct makes the measured and HBT-generated phase match quite well above 1kHz.

But what about the phase below 1kHz?.

Measured phase response at low-frequency end will be affected by the length and type of the FFT window used. Therefore, the phase will deviate from the minimum-phase trajectory. To avoid this issue, one could use really wide FFT windows (if the measurement environment allows it) of use close-mike techniques.

In summary, it is suggested, that the phase response on the figure below (blue line) is the accurate representation of minimum-phase response of the measured loudspeaker.





8. Conclusions

SPL can only be extracted accurately by IHBT, if the supplied phase is of minimum-phase type.

Between the phase attachment points, the IHBT is insensitive to attached phase (SPL) slopes, as the SPL does not change between attachment points.

Between the phase attachment points, the IHBT is quite sensitive to non minimum-phase phase response, behaviour, as the SPL does change quite significantly between attachment points.

The IHBT can be used as a “detector” to determine if the phase response is indeed the sought-after minimum-phase response.