Automated Methods For Minimum-Phase Extraction

There are two automated method for minimum-phase extraction implemented in this program. The first method is based on HBT algorithm, and is considered an unconstrained method. The second method, described previously, is based don IHBT algorithm, and includes constraining parameter, which helps the algorithm to reject false Error Minimum values.

Inverse Hilbert-Bode Transform (IHBT) was described in Chapter 16.3. Followed by it's applications in minimum-phase extractions in Chapter 16.4. This chapter offers a summary and extensive examples and explanation of both automated methods. You will find, that extracting minimum-phase response from measurements is not a simple process, and it does require decision making on your part. The goal here is to provide you with enough base line data, so that you can make an informed decision on minimum-phase response determination.

IMPORTANT: Once you activate either of the two methods, it's best to leave the computer alone, and let it run the extensive calculations till the completion.

Both methods are controlled from single dialogue box. The control box was described earlier, but because it has been modified to accommodate two methods, the individual controls are described below.

"Start HBT Method" – This button activates the unconstrained HBT algorithm.

"Start IHBT Method" – This button activates constrained IHBT method.

"**Constraint [dB]**" – This is the low-pass slope in dB/oct guiding the IHBT algorithm.
"**Sweep +5,+10..**" – Select this checkbox for automated increments of Constraint parameter. The algorithm will add fifteen 5dB increments to the selected Constraint [dB] value, and will search for a minimum error for each Constraint. This is the most comprehensive search for the minimum-phase response. Please be prepared, that this is really very long process, if you select this option.

"HBT Error Fcn" Use this checkbox to plot HBT error vs. frequency.

"**Measured Phase**" – Use this checkbox to plot MLS or ESS measured phase.

"**HBT Phase**" – Select this button to plot HBT-derived phase.

"**MLS ESS**" – Select which method was used in measuring the SPL/Phase response.

"**IHBT Error Fcn**" – Use this checkbox to plot IHBT error vs. frequency.

"Error Split [Hz]" – During error analysis, it is convenient to determine which frequency range is responsible for error growth. Typically, higher frequencies will be responsible for most of the error growth. The Error Split frequency determines the splitting point.

Automated Methods always start with a MLS or ESS measurements. The process of measuring a loudspeaker and selecting the correct starting FFT Windows position for automated algorithms was described before. Therefore, the remainder of this Chapter will focus on factors, that will help the operator make the correct decisions while extracting the minimum-phase response. The manual also offers some background factors that lead to development of this particular type of algorithms.

Next, we offer a number of real-life loudspeaker drivers run through the processes, and we discuss selection of critical starting parameters for the automated methods. The results are discussed and finally some conclusions are drawn.

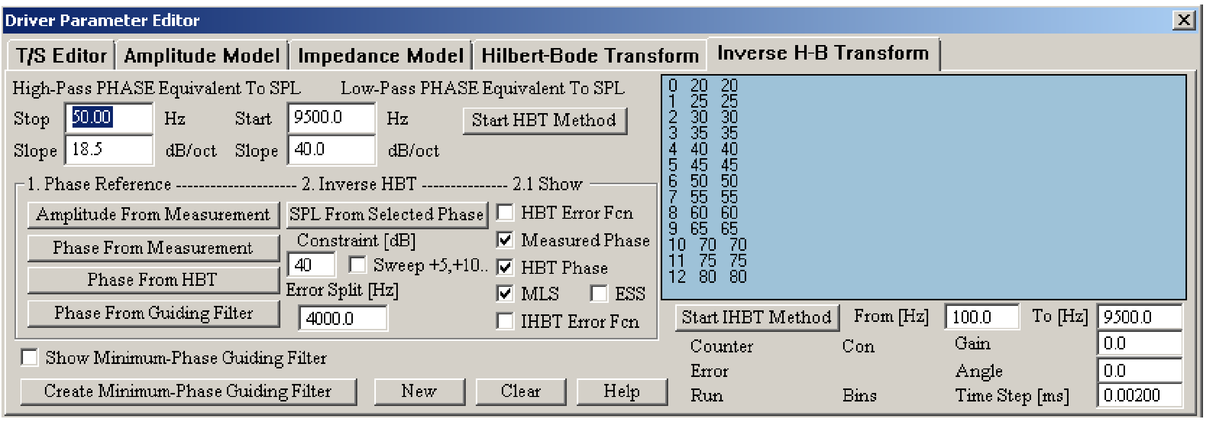


Figure 16.303. Minimum-Phase extraction control dialogue.

It it important to observe, that:

The “From [Hz]” frequency must be higher or equal to “Stop” frequency below
The “To [Hz]” frequency must be lower or equal to “Start” frequency below

Introduction

When the HBT Method was first introduced, about 20 years ago, there were attempts to implement the method for the purpose of extracting minimum-phase response from a measured SPL response. The HBT Method is based on Dr Bode’s integral formulation. To solve Bode’s integral as intended, one needs the knowledge of the function’s behaviour at infinity. This translates into simpler language as: need to know the asymptotic behaviour as the function approaches infinity – the asymptotic slopes. This implies, that the intended usage of the integral is to: (1) provide asymptotic slopes on the low-frequency side and high-frequency side and (2) then calculate minimum-phase response from known SPL data. The process will yield the correct result, as long as the whole function is of "minimum-phase" type.

Then, using the phase result as a template, one can adjust the excess phase in the measured result to match the calculated minimum-phase – thus obtain the desired SPL/Phase measurement with minimum-phase. Simple, if the slopes are known.

In practical terms, this process involved repeated adjustments of 4 parameters driving the HBT and also manipulating the delay time (excess phase) introduced inevitably in measurement process. The idea was that at some point during the adjustments, there will be nearly exact match between the measured phase and HBT-generated phase. At this point, the HBT adjustment parameters and the excess phase would define the minimum-phase match.

The Manual Method

If you wish to “optimize-by-hand” the process of finding the minimum-phase, then you are being asked to juggle 5 arbitrarily selected parameters: two attachment points, two asymptotic slopes and one excess phase data. During this process, and for each attempt, the user will have to eyeball two phase responses for the best match, while trying to remember how good was the match for the parameters selected before for other set of parameters. It is difficult to assess if you are moving in the right direction as there is no indication guiding the next step. Number of combinations is staggering, particularly when you start moving around the attachment points. Ambiguity of the eyeballing, lack of numerical information about the progress, tediousness and length of the process are just some of the drawbacks of the manual process.

Visual inspection can be very difficult. Figure 16.304 shows two phase responses: (1) 36dB/oct low-pass filter (**blue**) and (2) 30dB/oct low-pass filter with 20usec delay added to it (**green**). It is observable, that up to 10kHz, the two phase responses are nearly identical. They start to diverge rapidly above 10kHz. The operator would easily accept this result as perfect match – if the frequency range was limited to 10kHz. Then, the excess phase would never be determined.

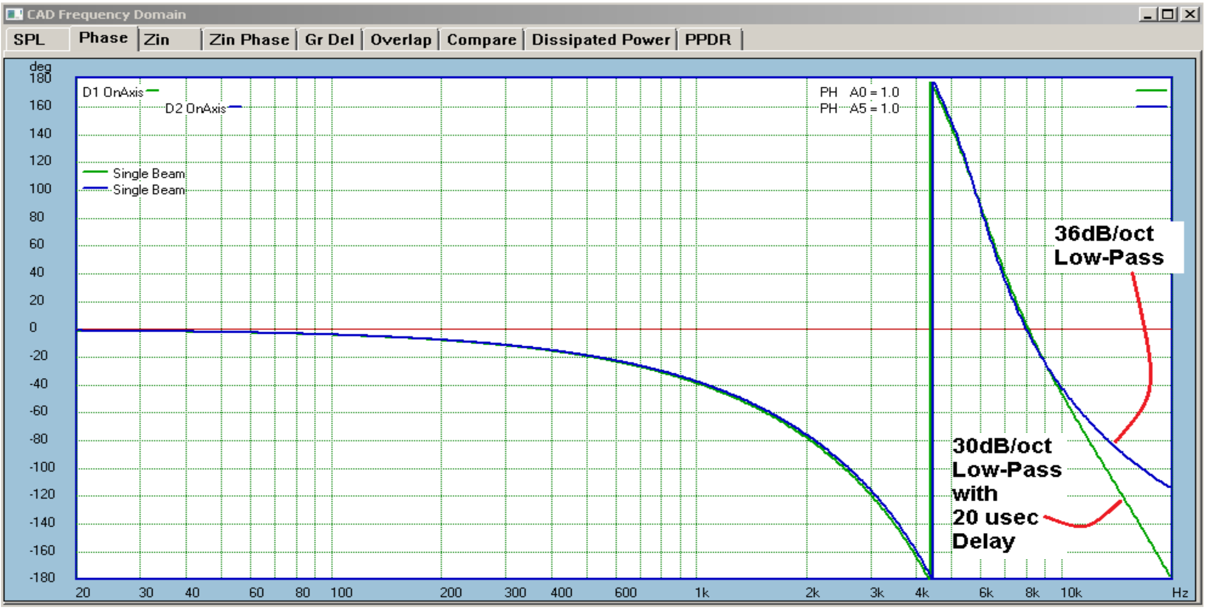


Figure 16.304. 36dB/oct LP filter (Blue) and 30dB/oct LP filter with 20usec delay.

The phase response of the 6kHz low-pass filer shown on Figure 16.304, would represent a typical mid-woofer, with frequency bandwidth up to 6 kHz and break-up region above that point. So, it would be a challenge to determine minimum-phase for such driver, given similarity of the phase responses up to 10kHz. When one relates this issue to loudspeakers in general, it would be prudent to examine the problem of phase matching, over **the widest possible frequency range** for three reasons, and one opposing.

1. The error between the measured phase and HBT-derived phase will manifest itself more strongly towards the higher frequencies. The error may rapidly increase for wider frequency spans.
2. The high-frequency end, may contain very valid measured data for SPL and phase, so purposefully eliminating this data would lead to diminished confidence in the overall results.
3. When calculating the cumulative error over some frequency range, the error will always be smaller for narrower frequency range. This will leave the operator with the impression, that the set of parameters corresponding the lower error, is the one that should be accepted.
4. On the other hand, attempts to include loudspeaker's break-up region in the process, may back-fire. The break-up region is known as non-minimum-phase region, and it will distort the overall results and accuracy.

Automated Method

The Automated Methods involve one (or more) minimum-seeking algorithms to manipulate available parameters in order to minimize the error between measured phase and the HBT-derived phase.

In short, for each optimization attempt, the whole measurement process is executed by including: (1) selection of windowing parameters and positioning of the FFT window, (2) FFT algorithm, (3) SPL/phase smoothing parameters, (4) Mike Cal file, (5) adding delays, and (6) minimum-phase extraction. Overall, it's not a simple process, even with automation.

The process starts with selecting the low-frequency and high-frequency attachment points (frequencies). Then, for each FFT window position, the measured phase is calculated using FFT algorithm. Next, the optimizing algorithm adjusts the low-end and high-end slopes for the minimum error between HBT-derived phase and measured phase.

The error is presented as numerical value, and can be later used to decide which set of HBT parameters and what position of the FFT window and delays delivered the smallest error – meaning, the best match between the measured phase and HBT generated phase. Then another set of attachment points is selected and the process is repeated.

Example of Tweeter 1

To illustrate the above concerns, we start with a tweeter driver. The SPL and phase responses are shown on Figure 2. The location of the FFT window was arbitrary, so the presented phase response is not the minimum-phase response.

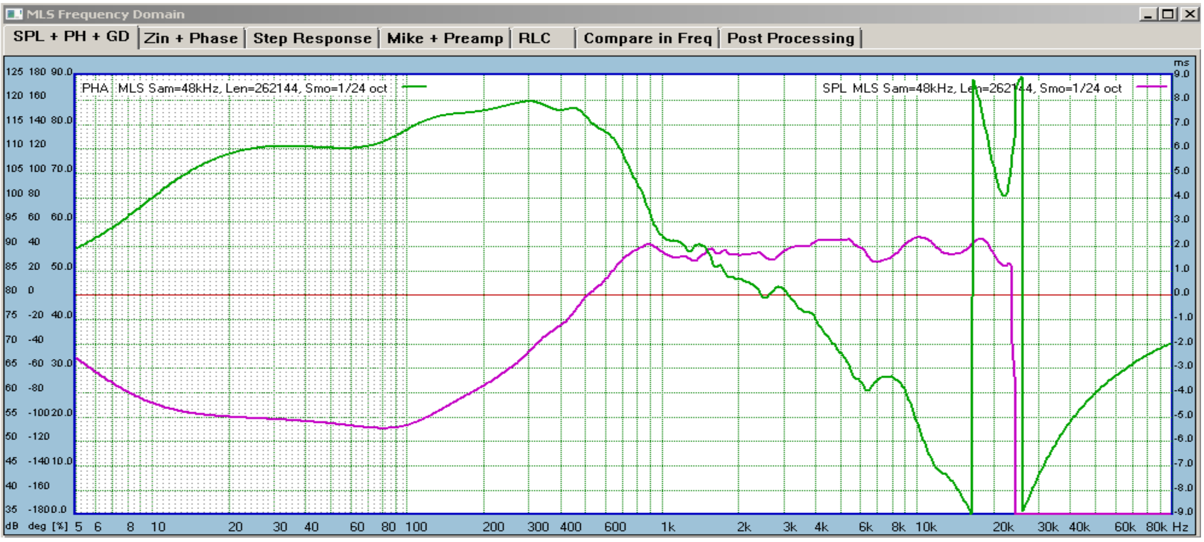


Figure 16.305. SPL/phase example – Tweeter 1 driver.

When different attachment point (frequency) is selected, the resulting phase response will change, therefore, the user is expected to adjust the high-pass or low-pass slopes to compensate for the change in the attachment point location.

Please note, that the excess phase has not changed, so re-adjusting the slopes should only compensate for the change in the attachment point.

This assumption has been tested using the Automated Method.

The results are tabulated below.

HBT LP Start	HBT HP Start	FFT Bin	Delay ms	LP Order	HP Order	Error
17000	400	90	0.002	8.94	11.99	330.61
18000	400	90	-0.002	14.38	12.01	366.99
19000	400	90	-0.002	14.75	11.99	399.04
20000	400	90	0	11.93	12.01	423.03
21000	400	90	0.002	8.97	12.01	464.91
22000	400	90	0	10.85	11.98	631.01

It is observable, that the algorithm has adjusted the low-pass and high-pass slopes for different attachment points, but is has also suggested different delays (Bin/Delay) – meaning different excess phases for each new attachment point. This will be evident in tabulated results for other drivers.

Recommended phase response for attachment point of 17000Hz is shown on Figure 16.306 below.

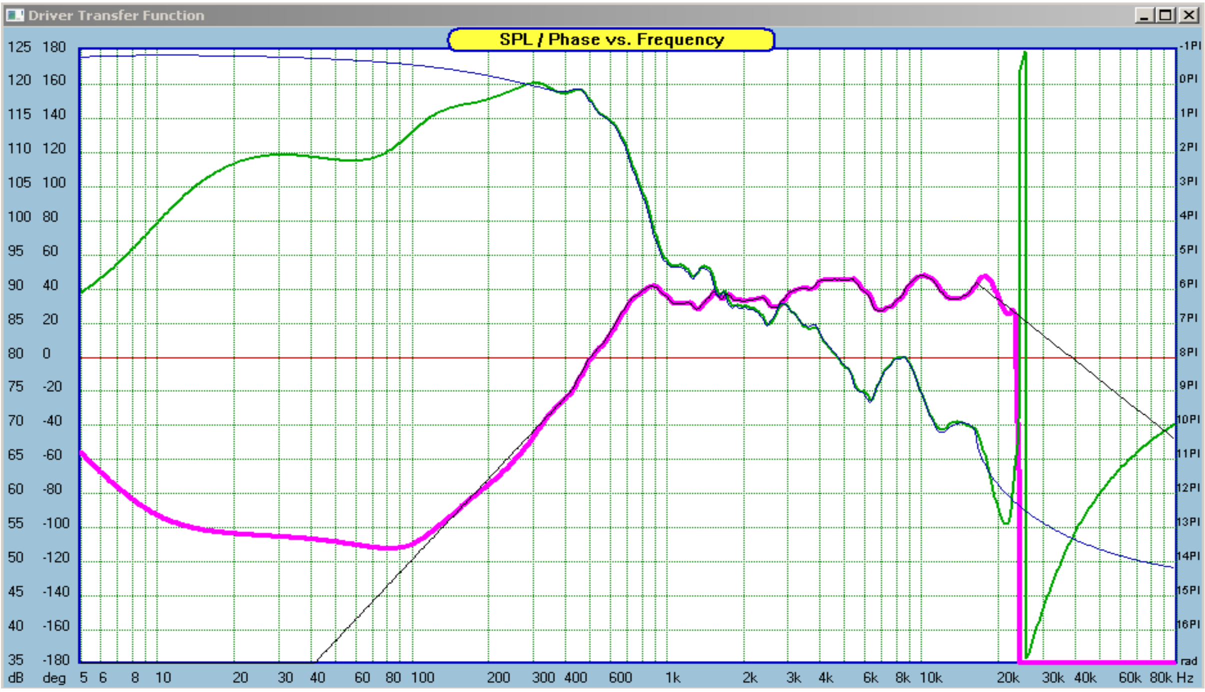


Figure 16.306. Phase response for the attachment point of 17000Hz.

It is evident on Figure 16.306, that the attachment point was selected incorrectly, and a 5kHz portion of usable SPL between 17kHz and 22kHz has been neglected in optimizations. The result is unnaturally shallow high-frequency slope of 8.94dB/oct and a-typical phase response. Interestingly, this attachment point resulted in the lowest Error – because of the narrow frequency range selected.

More acceptable phase response is shown on Figure 16.307 below. It was generated for the following HBT parameters:

HBT LP Start	HBT HP Start	FFT Bin	Delay ms	LP Order	HP Order
22000	410	89	0.006	28.42	11.96

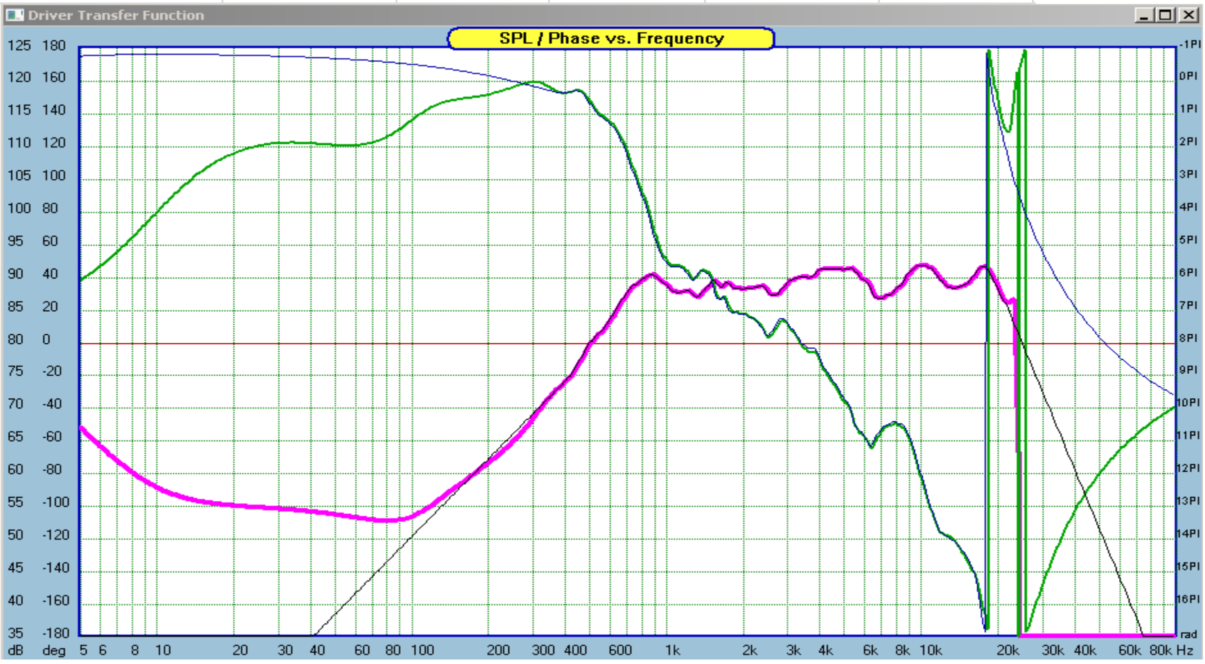


Figure 16.307. Minimum-phase phase response for Tweeter 1.

Example of Tweeter 2

Once again, the location of the FFT window was arbitrary, so the presented phase response is not the minimum-phase response. The SPL of Tweeter 2 is shown on Figure 16.308 below.

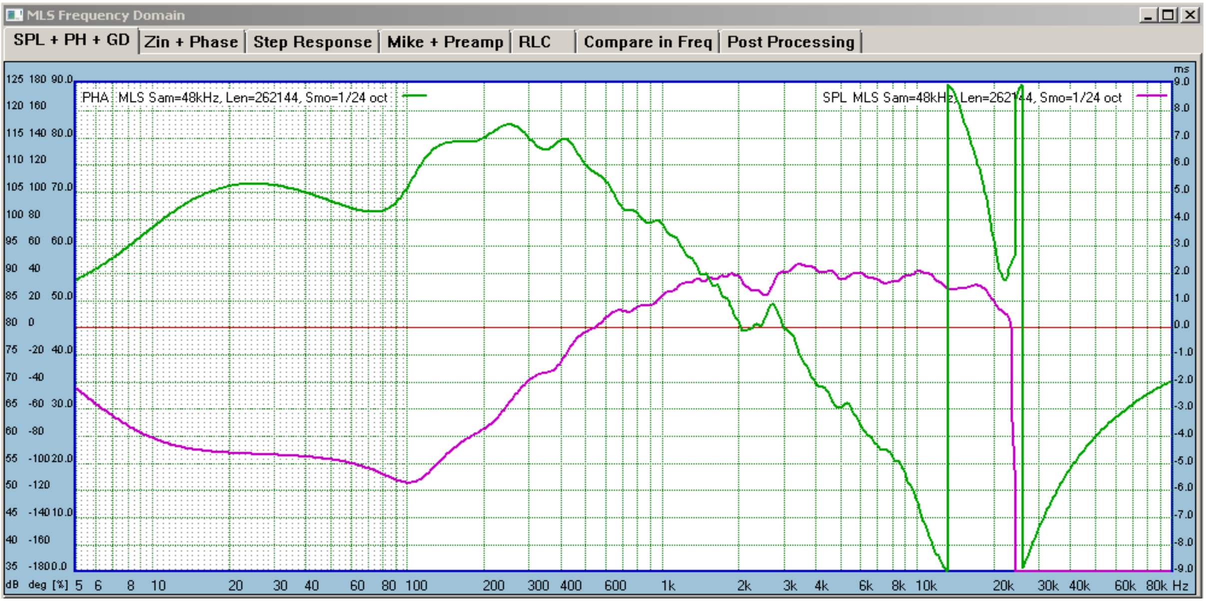


Figure 16.308. SPL/phase example – Tweeter 2 driver.

Tabulated responses for the Automated Method for different attachment point are shown below.

HBT LP Start	HBT HP Start	Bin	Delay	LP Order	HP Order	Error
17000	400	122	-0.002	15.65	11	298.76
18000	400	122	-0.004	18.76	11	358.33
19000	400	122	-0.004	19.29	11.01	420.82
20000	400	122	-0.004	19.35	11.01	433.61
21000	400	122	-0.006	21.49	10.99	494.71
22000	400	121	0.01	27.77	10.79	751.69

The problem repeats itself – the attachment point of 17000Hz results in phase response on Figure 16.309 below.

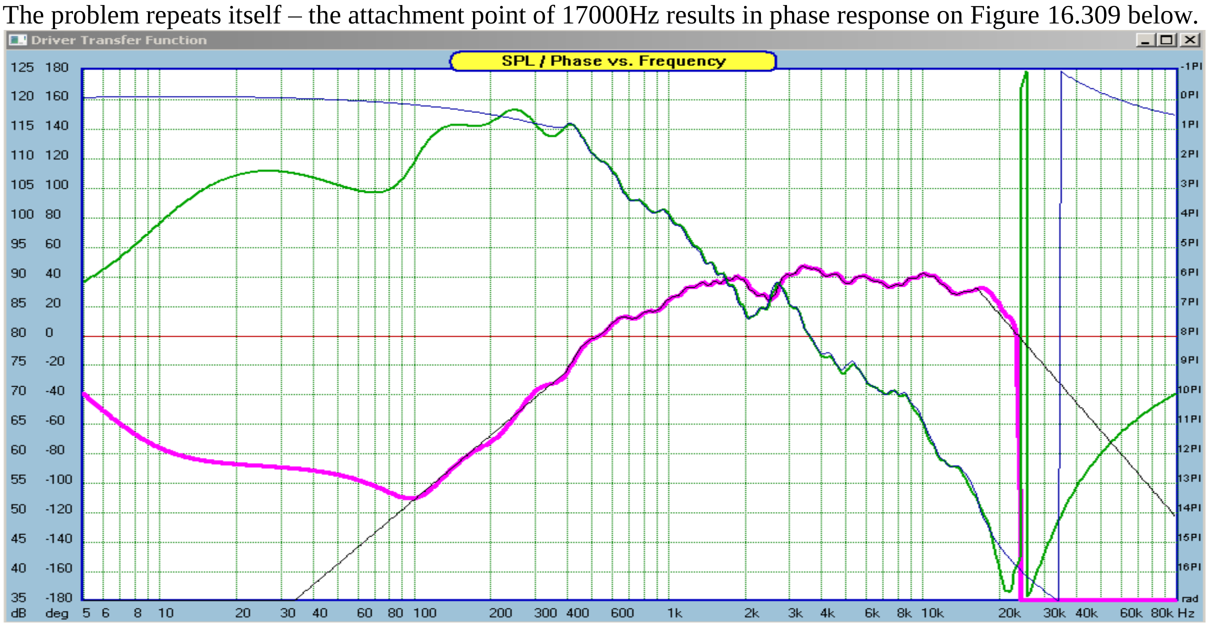


Figure 16.309. Phase response for the attachment point of 17000Hz.

More acceptable phase response is shown on Figure 16.310 below. It was generated for the following HBT parameters:

HBT LP Start	HBT HP Start	Bin	Delay	LP Order	HP Order
22000	410	121	0.004	34.87	10.74

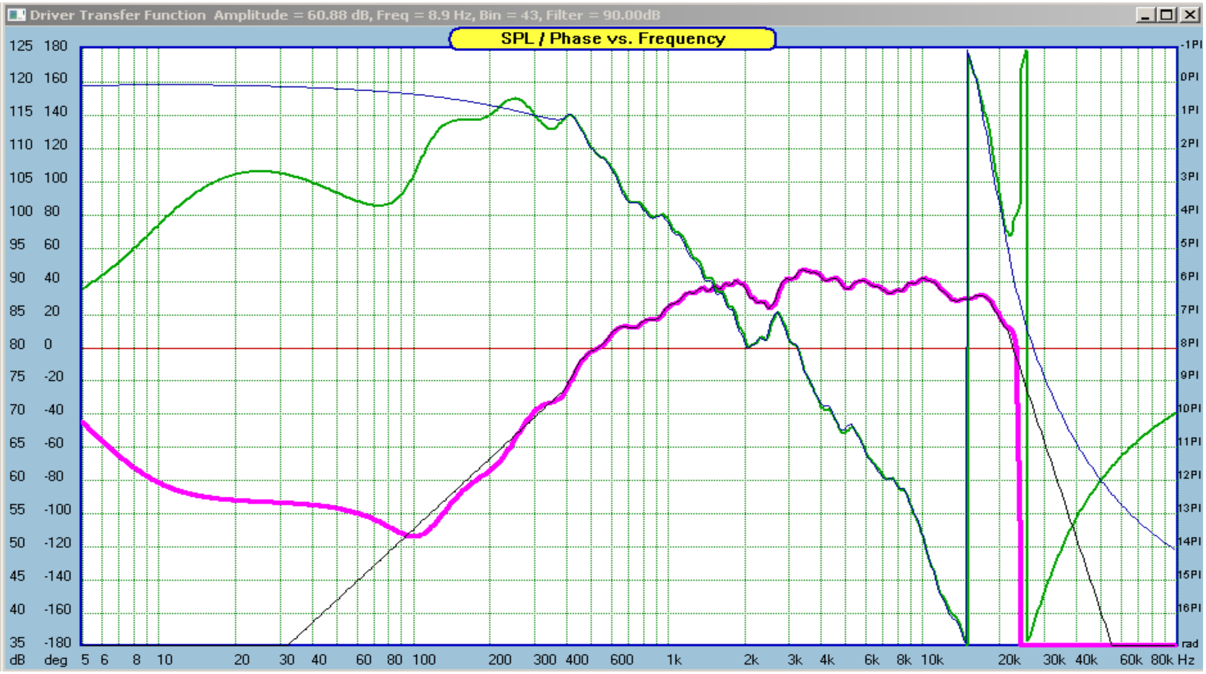


Figure 16.310. Minimum-phase phase response for Tweeter 2.

Example of Woofer 1

Once again, the location of the FFT window was arbitrary, so the presented phase response is not a minimum-phase response. The SPL of Woofer 1 is shown on Figure 16.311 below.

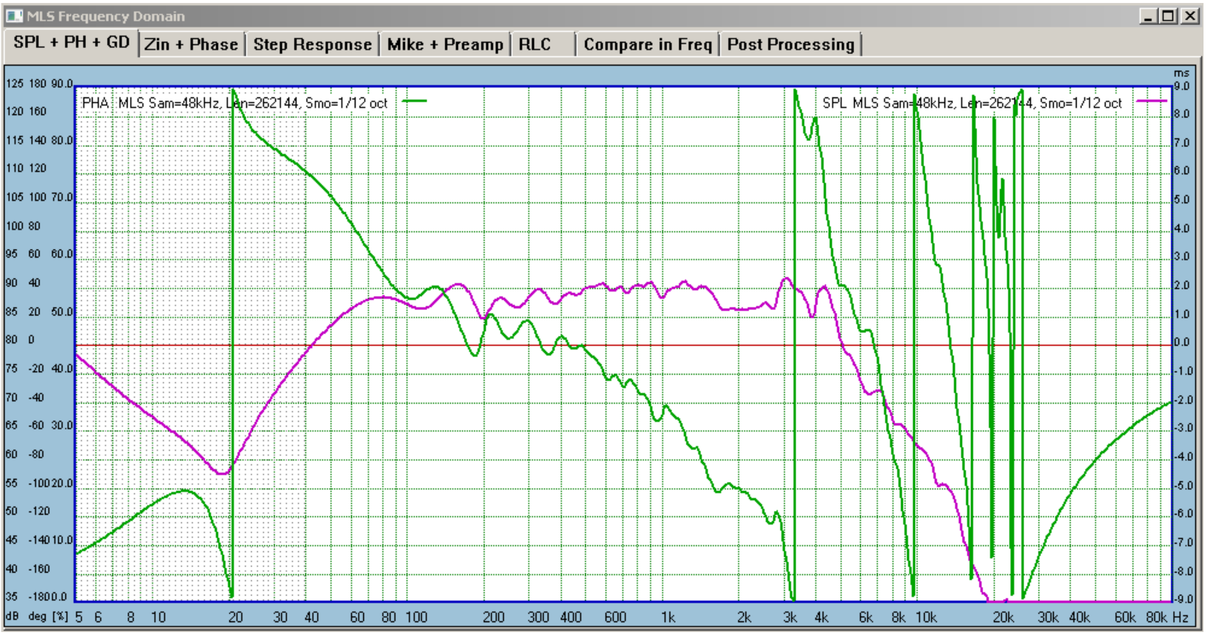


Figure 16.311. SPL/phase example – Woofer 1 driver.

Tabulated responses for the Automated Method for different attachment point are shown below.

HBT LP Start	HBT HP Start	Bin	Delay	LP Order	HP Order	Error
4000	50	127	0.008	12.5	9.56	486.69
5000	50	123	0.008	38.42	9.92	385.42
6000	50	124	0	36.08	9.95	542.29
7000	50	124	-0.006	42.36	9.97	718.35
8000	50	122	0.004	60.47	9.95	635.11
9000	50	120	0.002	88.43	9.93	586.06
10000	50	117	0.006	129.45	9.9	770.97

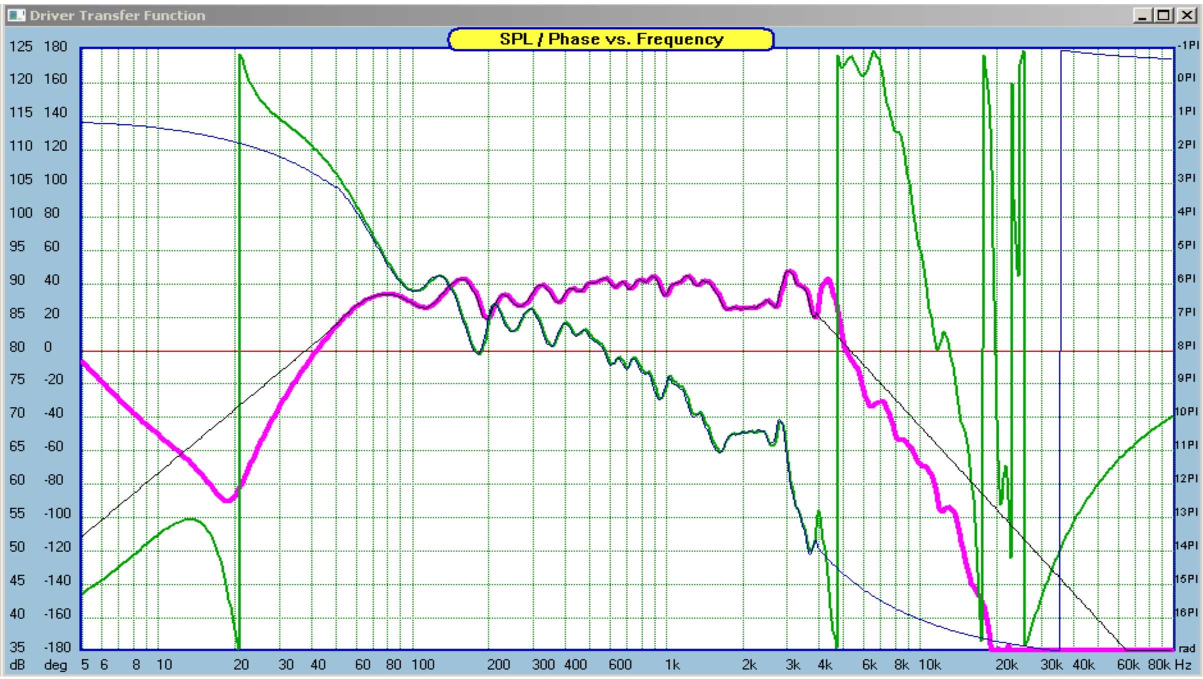


Figure 16.312. Phase response for the attachment point of 4000Hz.

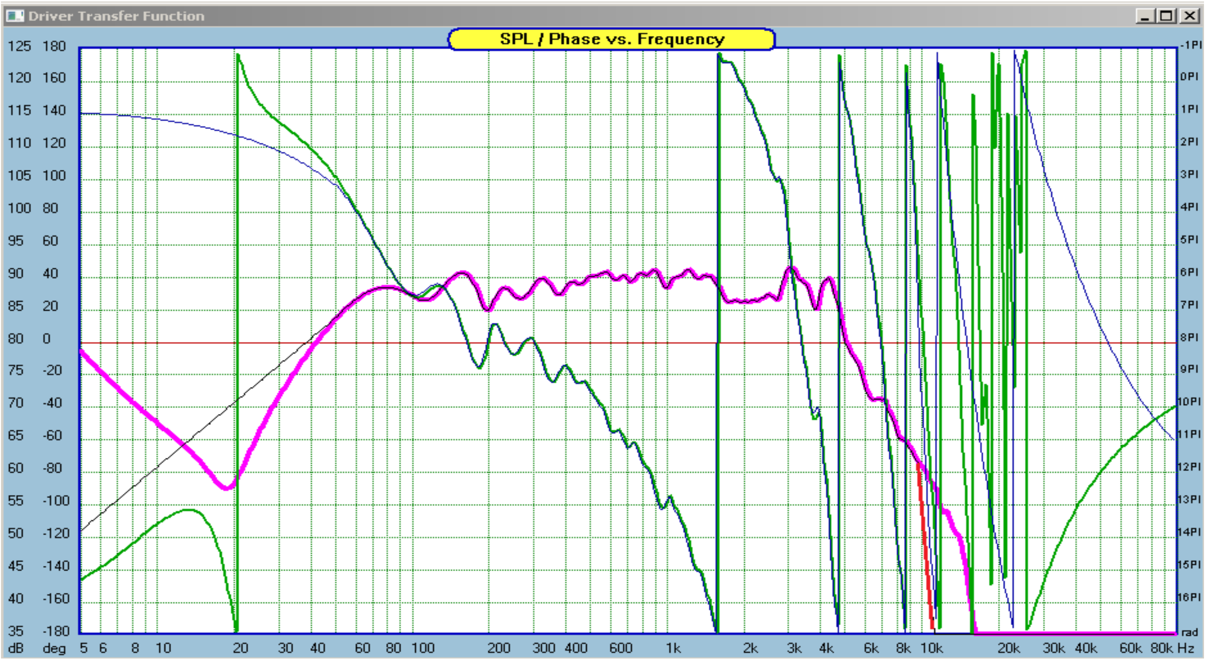


Figure 16.313. Phase response for the attachment point of 10000Hz.

In both instances, the matching between measured phase and HBT-derived phase is very good. But there are problems in both instances. Please note a completely unrealistic high-frequency slope of 129.45dB/oct (red) at 10kHz on Figure 10. This is plainly wrong, yet, it was recommended for the 10kHz attachment point.

More acceptable phase response is shown on Figure 16.314 below. It was generated for the following HBT parameters:

HBT LP Start	HBT HP Start	Bin	Delay	LP Order	HP Order
5000	50	124	-0.01	37.64	9.92

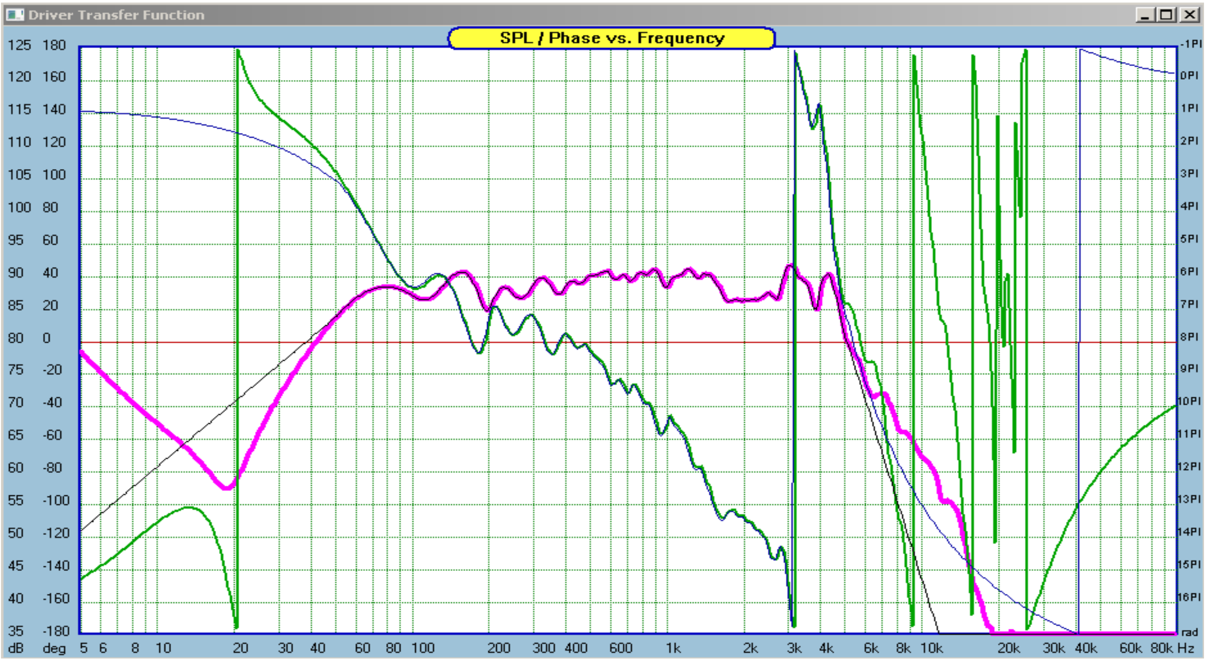


Figure 16.314. Minimum-phase phase response for Woofer 1.

Example of Woofer 2

Once again, the location of the FFT window was arbitrary, so the presented phase response is not a minimum-phase response. The SPL of Woofer 2 is shown on Figure 16.315 below.

Tabulated responses for the Automated Method for different attachment points are shown below.

HBT LP Start	HBT HP Start	Bin	Delay	LP Order	HP Order	Error	Bins	Error
5000	50	78	-0.012	24.87	15.74	2836.7	349	8.12808
6000	50	79	0	13.67	15.76	2939.15	362	8.1192
7000	50	79	-0.002	16.47	15.76	3004.13	374	8.03243
8000	50	78	-0.01	31.84	15.76	2947.25	384	7.67513
9000	50	77	0.004	35.21	15.73	3089.62	393	7.86163
9500	50	77	-0.002	38.34	15.72	3174.93	397	7.9973

It is interesting to notice, how close (+/-2.6%) the normalized error results are in the second “Error” column. These are all good phase matches, which would be difficult to discriminate visually.

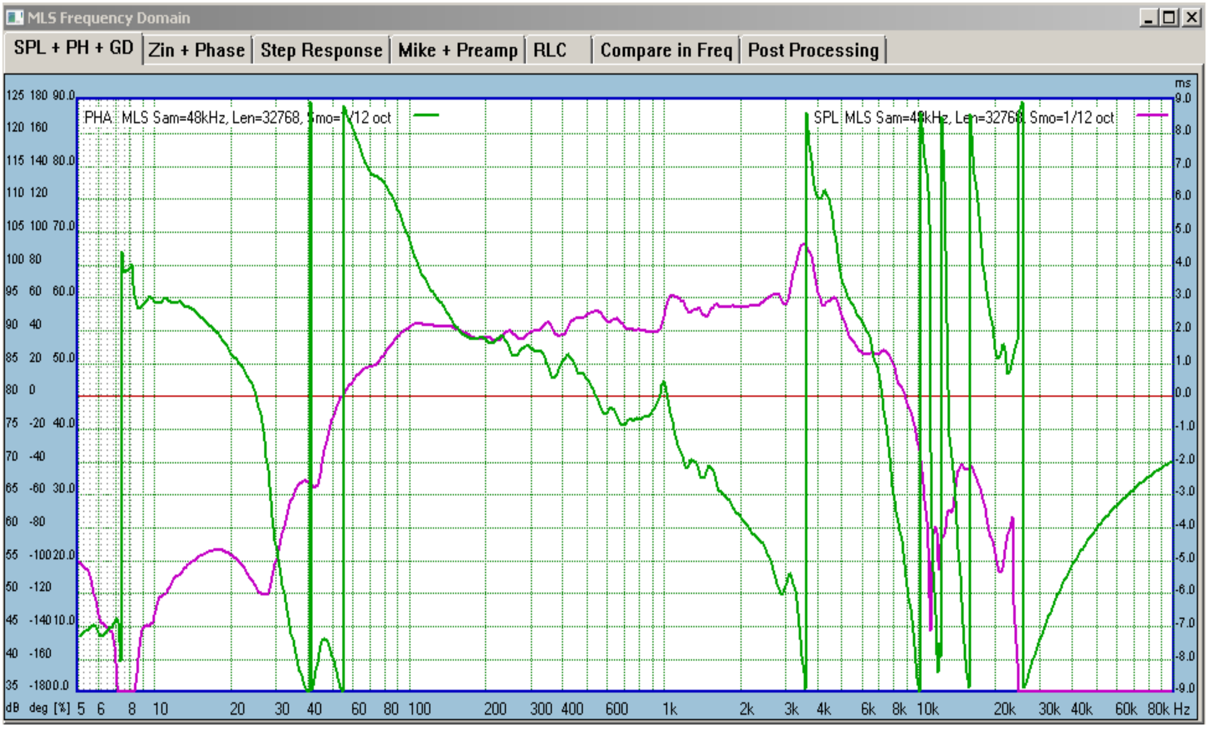


Figure 16.315. SPL/phase example – Woofer 2 driver.

Not only the Cumulative Error needs to be calculated, but the number of frequency bins also needs to be taken into account. Without this factor, the error results may favour the narrower HBT frequency bandwidth This is illustrated in the tabulated results above. The first “Error” column favours the 5000Hz attachment point. But when the number of frequency bins is accounted for using the “Bins” column, the 8000Hz attachment point would look better in the second “Error” column.

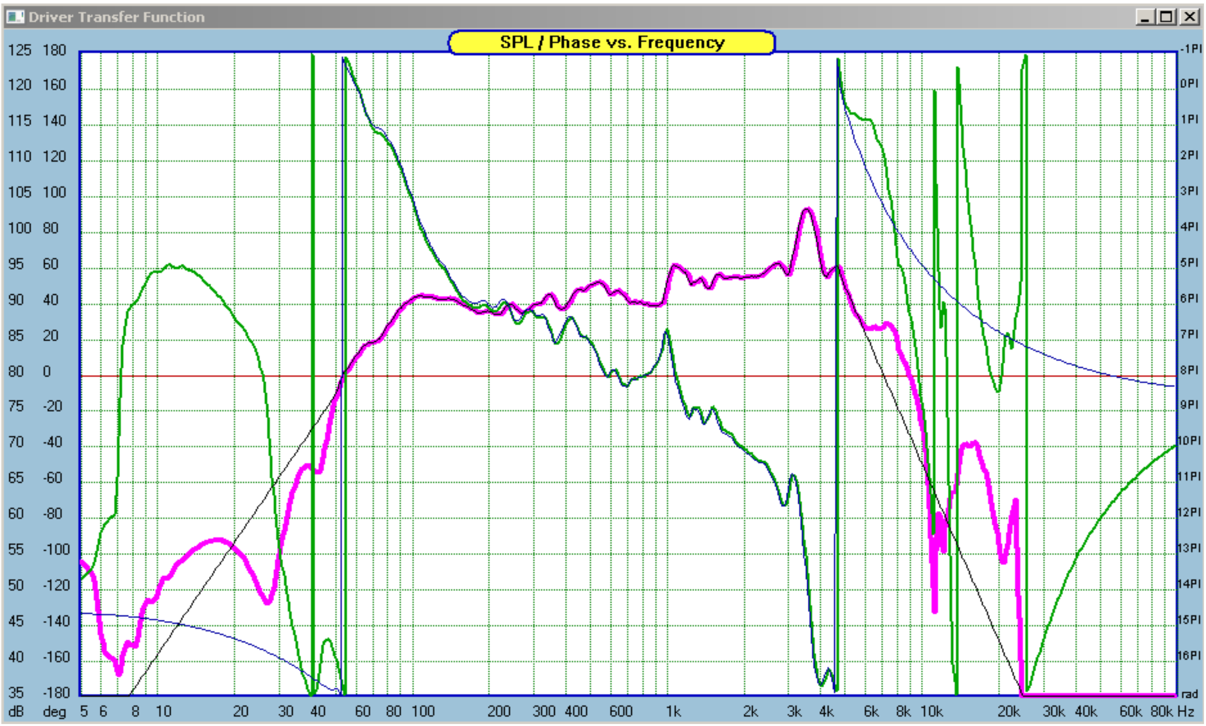


Figure 16.316. Phase response for the attachment point of 5000Hz.

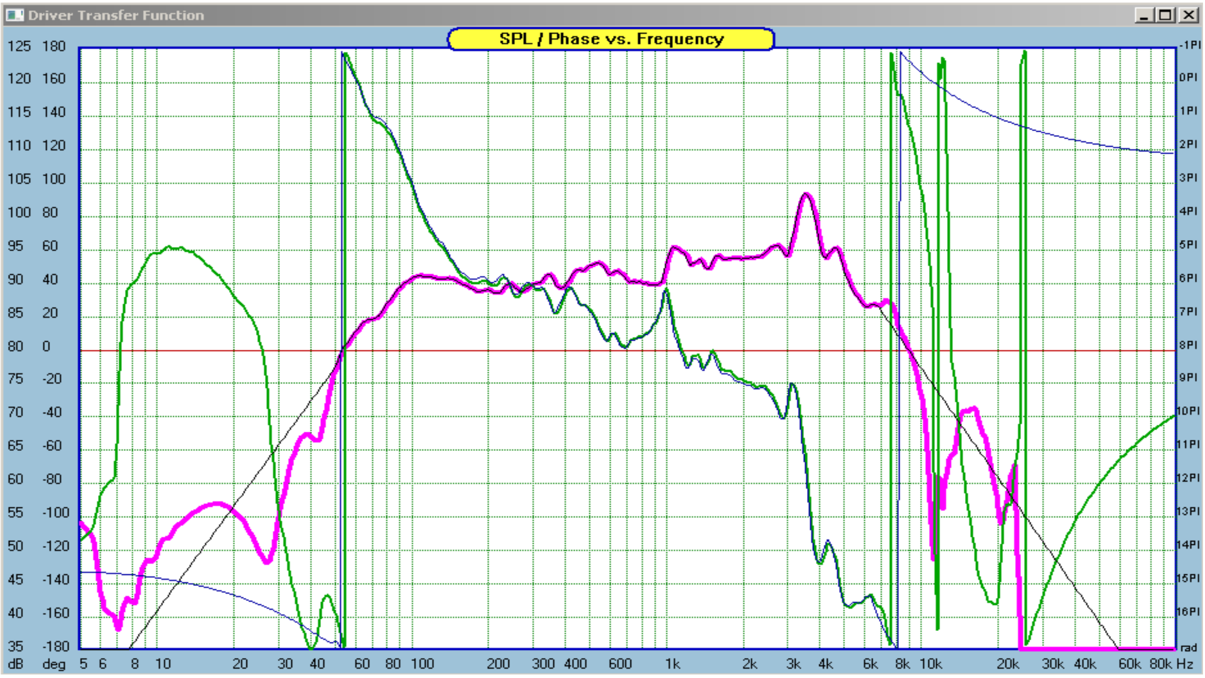


Figure 16.317. Phase response for the attachment point of 7000Hz.

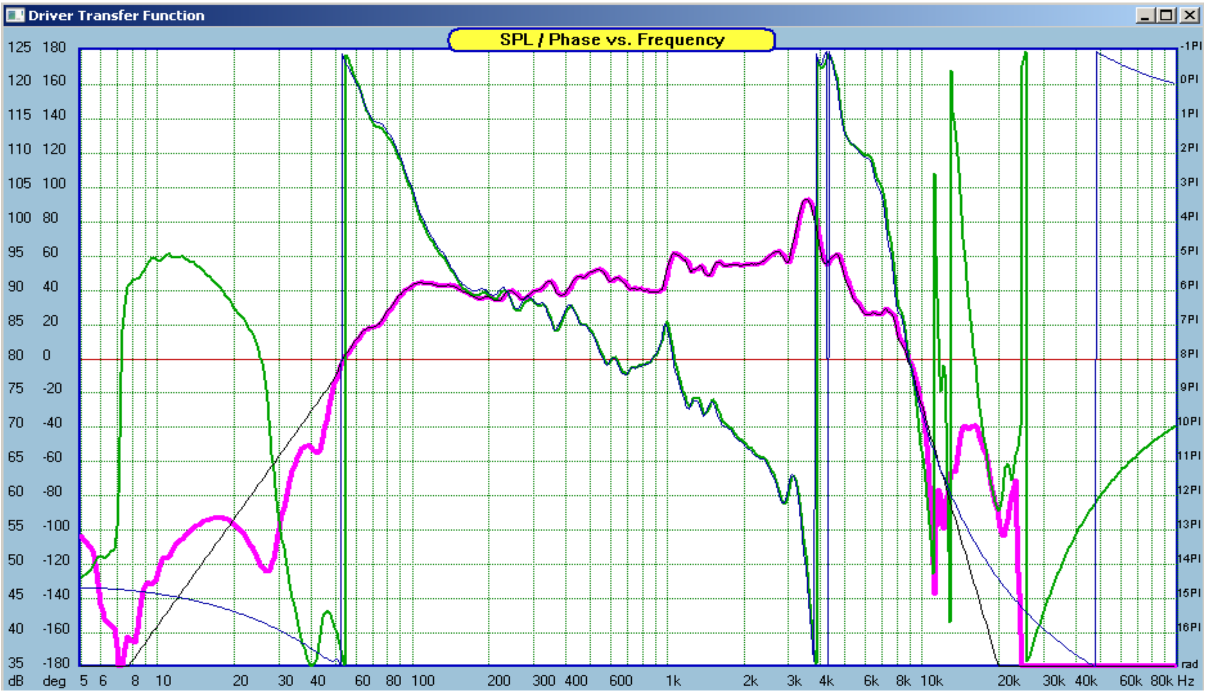


Figure 16.318. Phase response for the attachment point of 9500Hz.

It is observable, that the Cumulative Error (Error) increases as the HBT frequency range is increased. This is to be expected. This problem can be reduced by taking into account number of frequency bins between the attachment points.

More acceptable phase response is shown on Figure 16.319 below. It was generated for the following HBT parameters:

HBT LP Start	HBT HP Start	Bin	Delay	LP Order	HP Order
6000	100	77	0	27.18	15.67

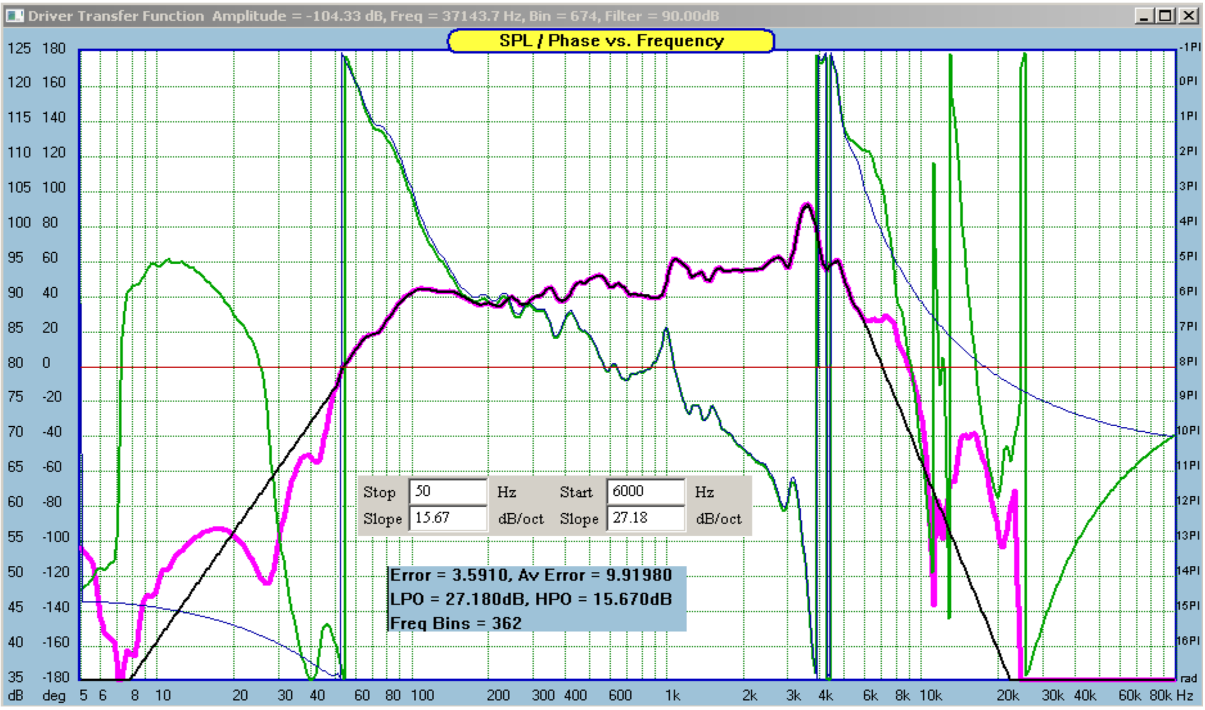


Figure 16.319. Minimum-phase phase response for Woofer 2.

Until this point, the discussion presented here highlights some of the issues manifesting themselves during minimum-phase extraction attempts using “manual HBT phase matching” techniques and “automated HBT phase matching techniques”. The processes are difficult.

The manual process relies on visual inspection of the two phase responses and often can be very challenging, as shown of Figure 16.304, and illustrated by a number of other examples. The automated process delivers Cumulative Error value, and accounts for bandwidth, allowing the user to make informed judgment on the quality of the match.

A number of phase matching examples were presented already, where the automated HBT process delivered good indication of matching, and this was supported by presenting the corresponding phase plots. Some were close to the minimum-phase data we were searching for.

Several problems attributed to the selection of the attachment points were also discussed. Selection of attachment points can be challenging as well, as some of them may have to be discarded. Examples presented above would indicate, that testing strategically selected 2-3 attachment points may be sufficient to get good quality data without clogging the picture too much.

The Automated HBT process was specifically designed to include all components of the measurement process. This is because each component can minutely contribute to the final accumulated error result. It is a genuine minimum-phase extraction from measurements.

It was also observed, that extreme cases of phase match can still be recommended by the manual and automated algorithms. The process of eliminating those cases was based on SPL curve, rather than phase responses. For instance, on Figure 16.306, the phase response (thin blue curve) maybe perfectly acceptable, but the corresponding amplitude response, rolling-off at -8.9dB/oct (thin black curve) shows, that loudspeaker could not have this SPL response. Also, on Figure 16.313, the phase response is technically acceptable, but the amplitude response rolling off at -126dB/oct is unrealistic.

This is an interesting observation. Here, we are working on determination of the minimum-phase responses, but we are still accounting for corresponding SPL, as the means for discriminating between acceptable and non-acceptable phase responses.

Perhaps a better designed minimum-phase extraction process needs to include the following:

1. Numerical error indication to allow for un-bias selection of phase responses.
2. Process needs to be automated to allow for mathematical precision into the process instead of visual inspection.
3. Optimization for the smallest error needs to be “constrained”. In this case, the algorithm would be guided into the area, where the resulting phase response is not of the extreme type, even if the error is not the smallest.
4. As discussed above, the constraint may need to be based on SPL curve, rather than phase response.
5. All components and activities associated with extracting minimum-phase response from measurement need to be included in the phase extraction process.

One issue was evident while examining tabulated responses of all drivers. There was a dependence of the excess phase calculations on the location of the attachment points. This is generally undesirable, because the excess phase is a property of the measurement system distances and remains constant. The excess phase does not change in the fixed system, and the “Bin” and “Delay” values should remain constant. Consequently, one would assume, that changing the attachment points would require changing of the slopes, but the excess phase would remind constant. Tabulated results indeed show, that asymptotic slopes will change, but excess phase will too. The change in excess phase is of moderate size, and often may not present itself as a major problem. This problem will be elaborated upon in discussion below.

Constraint in Automated IHBT Method

The concept of constrained optimization was introduced before. In this section, we will examine this aspect in more details. We now return to the tweeter driver presented in the previous discussions.

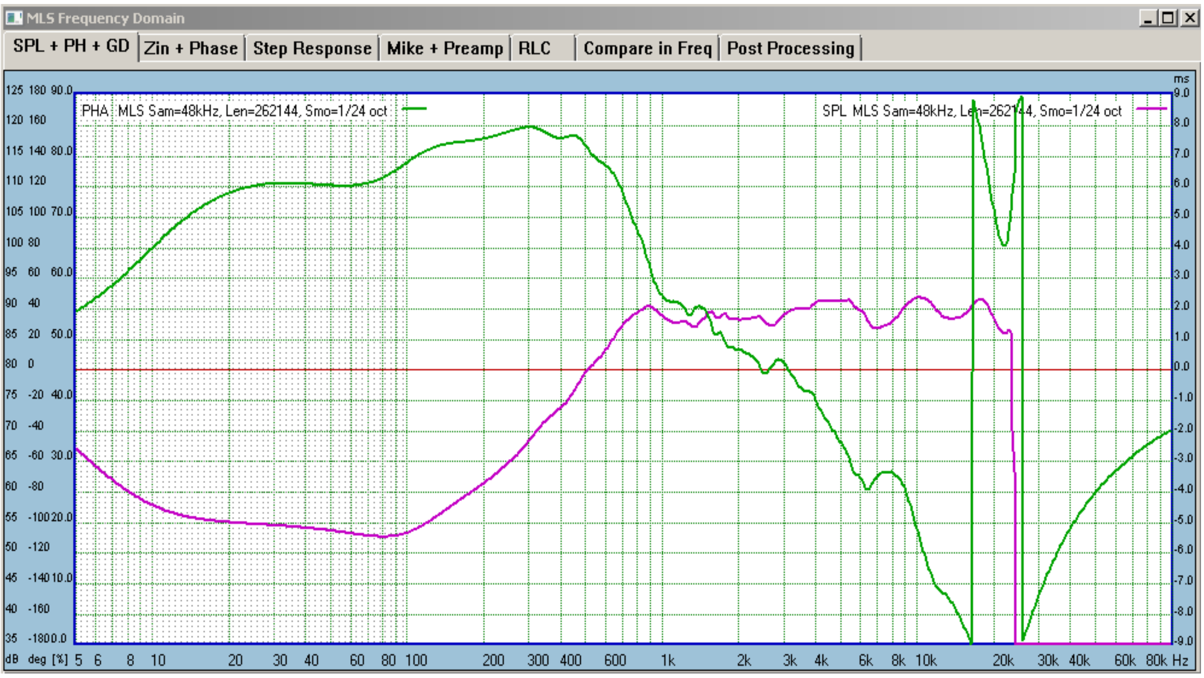


Figure 16.320. Example of SPL/Phase response of a tweeter driver.

Please note, that there is virtually no SPL/Phase data above 22kHz. Still, we should run the minimum-phase extraction over several possible attachment points and examine the results.

		Driver's Name		Jaycar_DT25_25cm_dist.res				
Start Bin =	85	Smooth =	1/24dB/oct		Window =	10ms		
Start HBT	400Hz	12dB	22000Hz	40dB				
HBT LP Start	HBT HP Start	FFT Bin	Delay ms	LP Order	HP Order	Error	Bins	Error
17000	400	90	0.002	8.94	11.99	330.558	284	1.16394
18000	400	90	-0.002	14.38	12	366.937	289	1.26968
19000	400	90	-0.002	14.75	11.99	398.982	293	1.36171
20000	400	90	0	11.93	12	422.975	297	1.42416
21000	400	90	0.002	8.97	12	464.848	300	1.54949
22000	400	90	0	10.85	11.99	630.953	304	2.0755
23000	400	90	-0.004	16.44	11.97	759.751	307	2.47476

Frequency responses of Tweeter 1 shown on Figure 16.306, that frequency range 17-20kHz will meaningfully contribute to the accuracy of the process, and 22kHz frequency may be too close to the limit of reliable data. Therefore, 21kHz attachment point is selected. It is observable, that low-pass slope attached at 21000Hz is not doing its job properly. Resulting phase response is too shallow. See Figure 16.321 below.

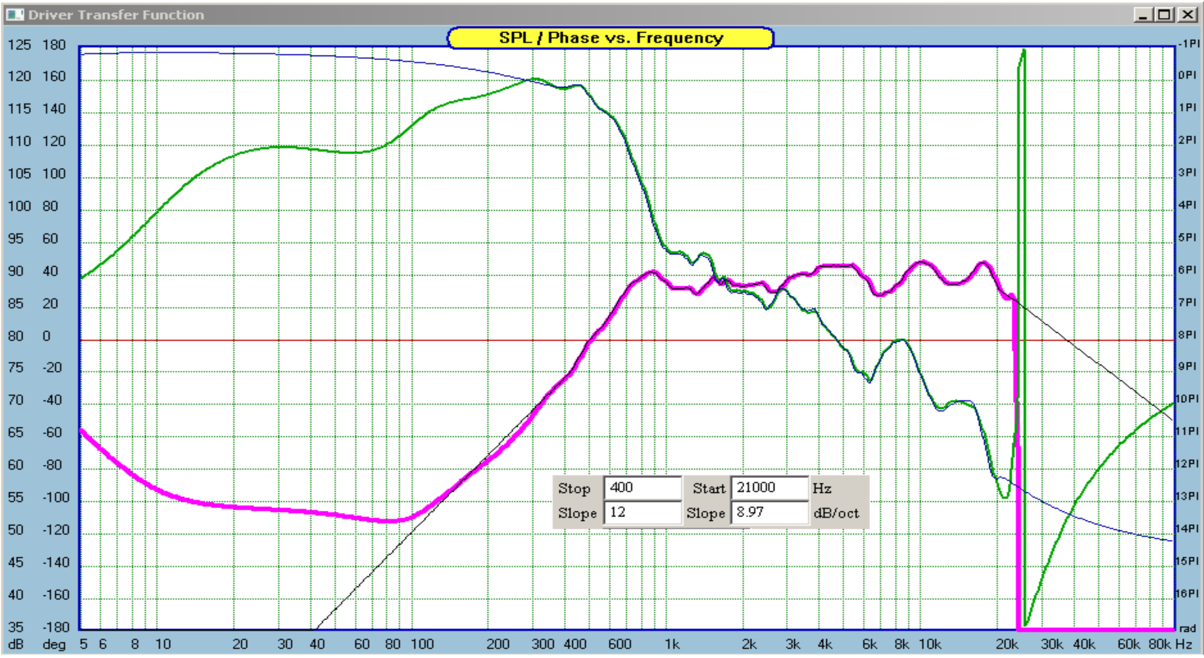


Figure 16.321. Shallow phase response.

Constrained IHBT optimization at 21000Hz leads to the following result.

								IHBT	Normalized
IHBT LP Start	IHBT HP Start	FFT Bin	Delay ms	LP Order	HP Order	Error	Bins	Error	Error
17000	400	90	-0.008	18.06	11.91	670.741	284	9.95	2.36176
18000	400	90	-0.008	20.2	11.94	486.404	289	10.07	1.68306
19000	400	90	-0.01	22.94	11.94	684.873	293	11.17	2.33745
20000	400	90	-0.01	22.72	11.91	979.844	297	14.25	3.29914
21000	400	89	0.01	23.43	12.02	1243.452	300	18.88	4.14484
22000	400	89	0.01	23.69	12.01	1185.611	304	21.66	3.90004
23000	400	90	-0.006	18.9	11.95	759.968	307	19.45	2.47547

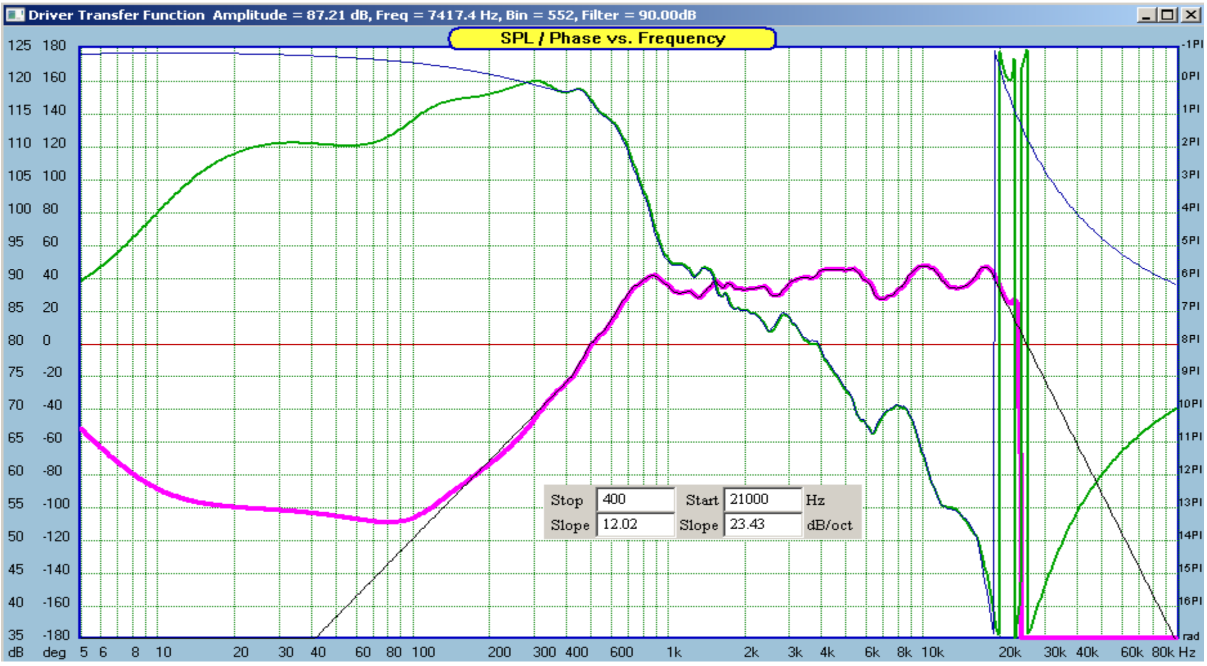


Figure 16.322. Phase response obtained using constrained optimization.

Next driver is a woofer driver.

Examination of the frequency response would indicate problematic SPL response above 5kHz.

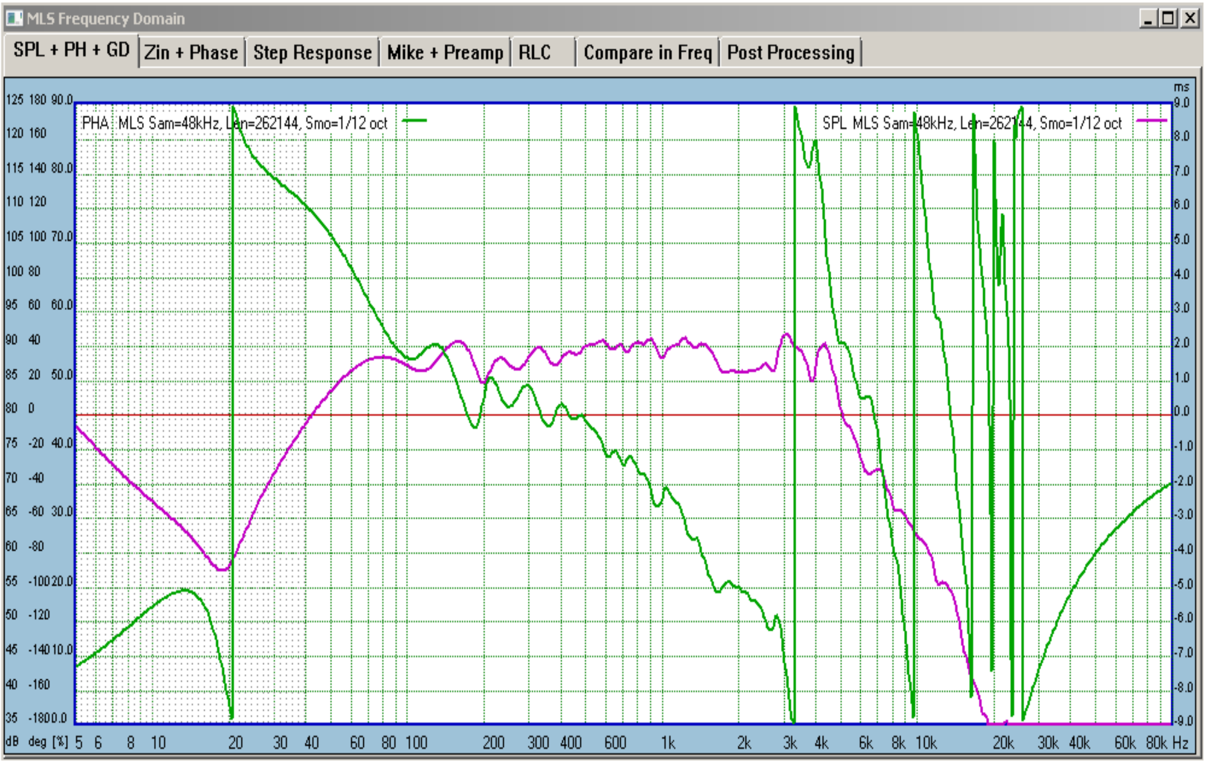


Figure 16.323. Example of SPL/Phase response of a woofer driver

		Driver's Name		Rear_Speaker_50cm.res				
HBT Method								
Start Bin =	119	Smooth =	1/12 dB/oct		Window =	25ms		
Start HBT	50Hz	18.5	4000	40				
						After HBT		Normalized
HBT LP Start	HBT HP Start	Bin	Delay	LP Order	HP Order	Error	Bins	Error
4000	50	127	0.008	12.5	9.96	486.676	332	1.46589
5000	50	123	0.008	38.42	9.92	385.908	349	1.10575
6000	50	124	0	36.08	9.95	542.306	362	1.49808
7000	50	124	-0.006	42.36	9.96	718.29	374	1.92056
8000	50	122	0.004	60.48	9.95	635.101	384	1.65391
9000	50	120	0.002	88.43	9.92	586.1	393	1.49135
10000	50	117	0.006	129.45	9.9	770.884	401	1.9224

Unconstrained HBT optimization is shown on Figure 16.324 below. If it wasn't for the tabulated Errors, it would be difficult to visually determine the best phase match. Even so, the Normalized Errors are not far from each other between 6000Hz attachment point, and 10000Hz result..

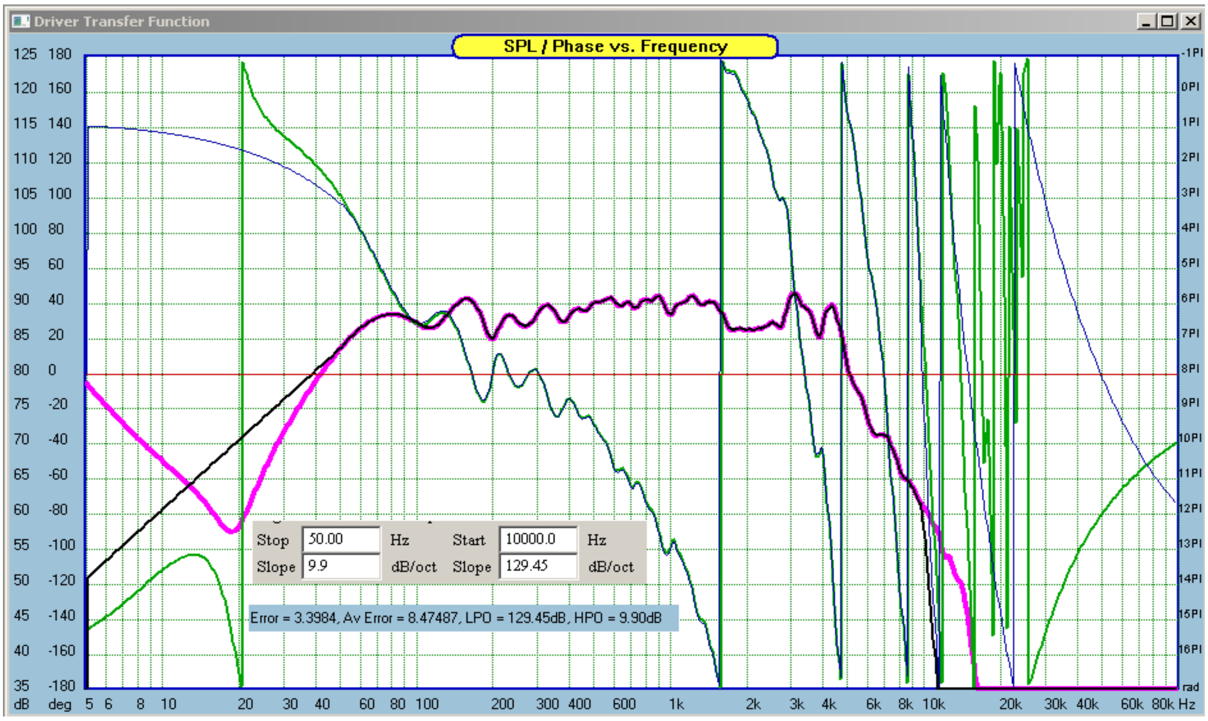


Figure 16.324. Example of visually very good phase match at 10kHz attachment point.

Compare the observations from above, to the "Constrained IHBT" optimization results tabulated below. The Normalized Error values increase markedly with the changes in the position of the attachment points. The Error at 10000Hz if almost **40 times higher** than at 5000Hz.

						After HBT		IHBT	Normalized
IHBT LP Start	IHBT HP Start	Bin	Delay	LP Order	HP Order	Error	Bins	Error	Error
4000	100	126	-0.01	21.05	9.9	781.05	332	9.83	2.35256
5000	100	123	0.008	38.42	9.92	385.908	349	9.48	1.10575
6000	100	124	0	36.08	9.95	542.306	362	11.64	1.49808
7000	100	124	0.01	36.32	9.99	861.57	374	16.55	2.30366
8000	100	124	-0.002	45.13	10.02	1598.38	384	18.78	4.16245
9000	100	123	0.008	55.55	10.06	5150.94	393	37.4	13.1067
10000	100	123	-0.004	68.23	10.12	16123.02	401	114.57	40.207

Figure 16.324, showing the phase match at 10kHz attachment point, is a good example why we need numerical indication of the Cumulative Error, preferably normalized to the number of data bins used. Visually, there is nothing wrong with this phase match, and even numerically, the Normalized Error at 10kHz is 1.92.

The results tabulated above for the constrained IHBT Method show rapid increase in Error for higher attachment points.

Finally, the results for the preferred 5000kHz attachment point are the same for both methods. This is to be expected, as both methods work on same principles. Automation just makes things quicker and more accurate. Please see Figure 16.325 and Figure 16.326.

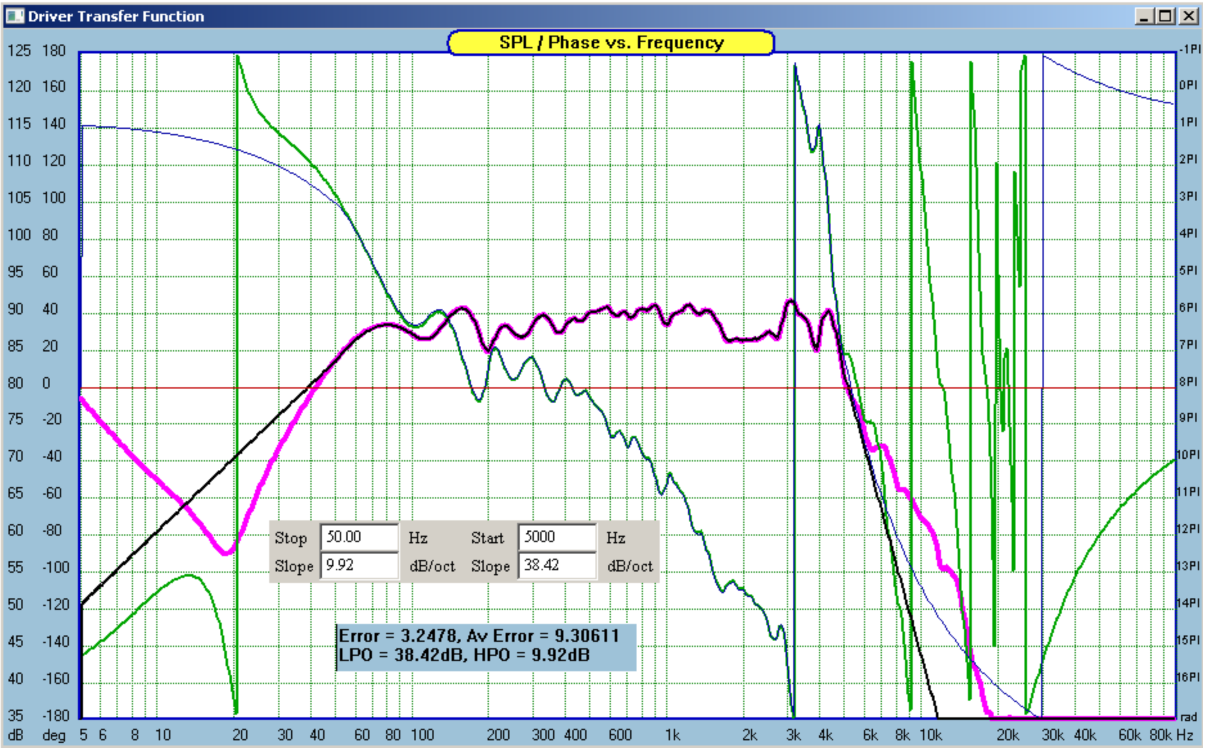


Figure 16.325. Optimum attachment point at 5000kHz

For 5kHz attachment point, the results from constrained IHBT and unconstrained HBT are identical. The 5kHz attachment point is quite safe to select.

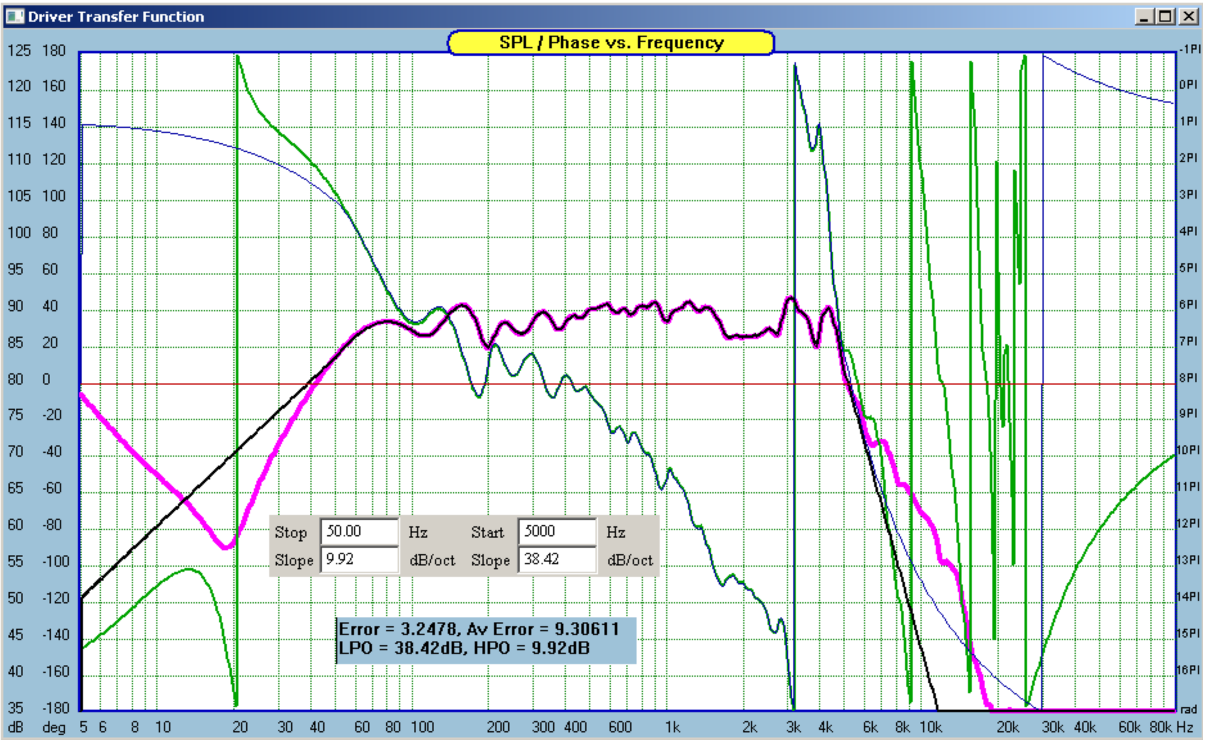


Figure 16.326. Optimum attachment point at 5000kHz

With the next tweeter driver, the situation is similar to the first tweeter. We observe, that frequency range 17-20kHz will meaningfully contribute to the accuracy of the process, and 22kHz frequency is too close to the limit of reliable data. Therefore, 21kHz attachment point is selected again.

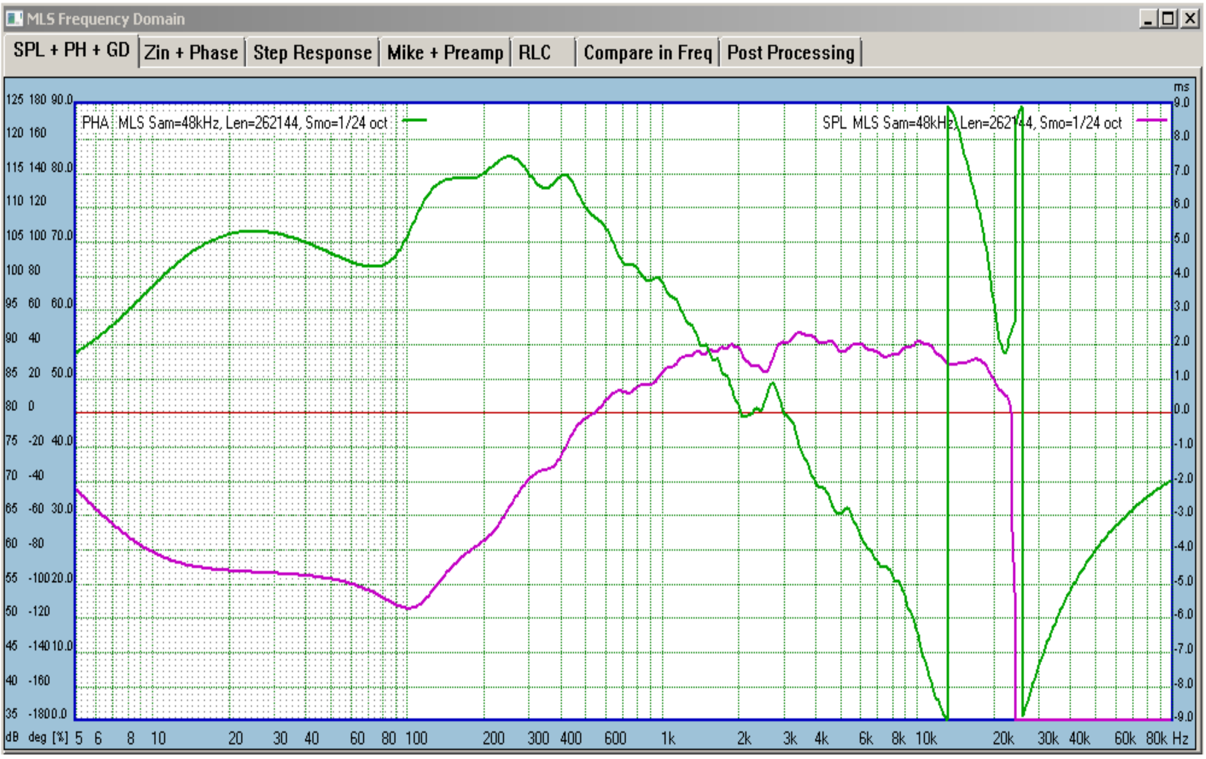


Figure 16.327. Example of SPL/Phase response of a tweeter driver.

Unconstrained optimization using HBT method yelds the attachment-dependant responses tabulated below

		Driver's Name		Front_Left_Speaker		Tweeter_50cm.res		
Start Bin =	119	Smooth = 1/24dB/oct			Window = 10ms			
Start HBT	400Hz	12dB	22000Hz	40dB				
								Normalized
HBT LP Start	HBT HP Start	Bin	Delay	LP Order	HP Order	Error	Bins	Error
17000	400	122	-0.002	15.65	10.99	298.285	284	1.0503
18000	400	122	-0.004	18.75	11.01	358.422	289	1.24021
19000	400	122	-0.004	19.3	11.01	420.824	293	1.43626
20000	400	122	-0.004	19.34	11.01	433.59	297	1.4599
21000	400	122	-0.006	21.5	10.99	494.438	300	1.64813
22000	400	121	0.01	27.77	10.81	751.727	304	2.47279
23000	400	121	0.006	33.57	10.78	832.637	307	2.71217

Please note, that the SPL response starts to show dropping tendency towards higher frequency. This is valuable information for all employed methods, and results in more accurate phase response determination.

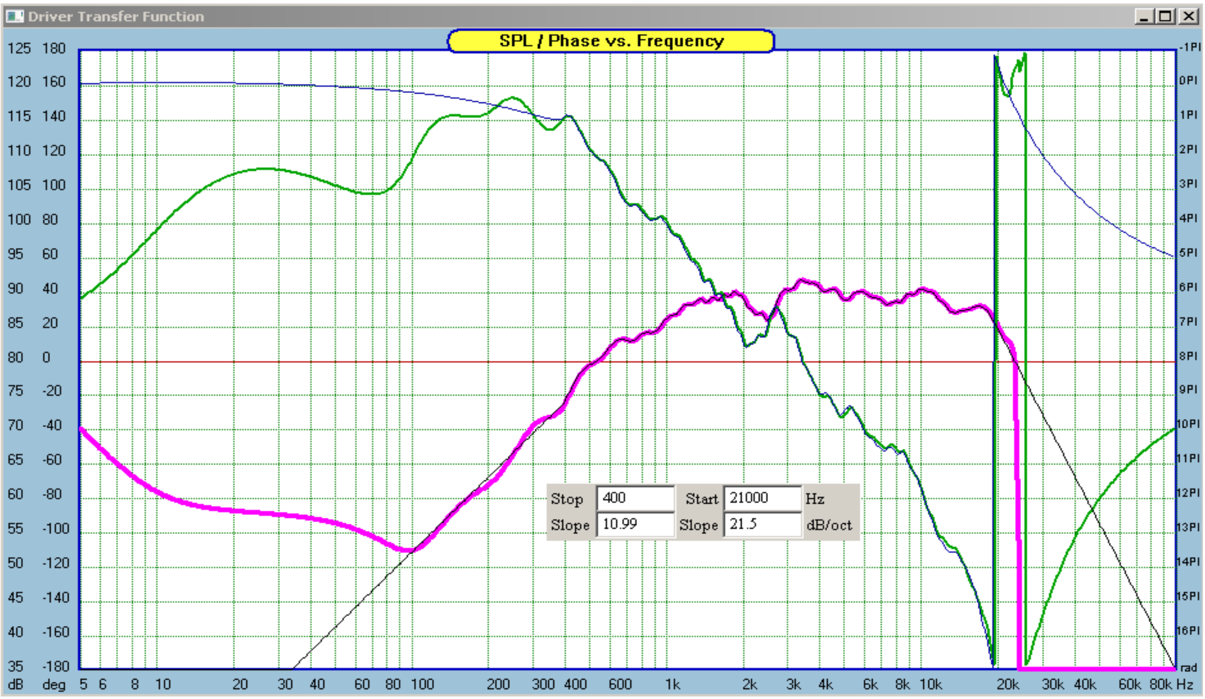


Figure 16.328. Phase response from unconstrained HBT method.

Constrained optimizations are tabulated below. Please note, that 22kHz Error is lower than 21kHz Error (last column). It was therefore decided to use 22kHz attachment point.

17000	400	122	-0.01	22.94	10.93	479.858	284	8.86	1.68964
18000	400	122	-0.008	22.64	10.97	393.292	289	10.28	1.36087
19000	400	122	-0.008	23.39	10.97	488.926	293	11.13	1.66869
20000	400	121	0.01	26.66	10.77	736.668	297	14.23	2.48036
21000	400	121	0.008	29.09	10.76	840.937	300	17.97	2.80312
22000	400	121	0.008	30.14	10.78	759.795	304	18.25	2.49933
23000	400	122	-0.008	25.03	11.02	882.543	307	24.56	2.87473

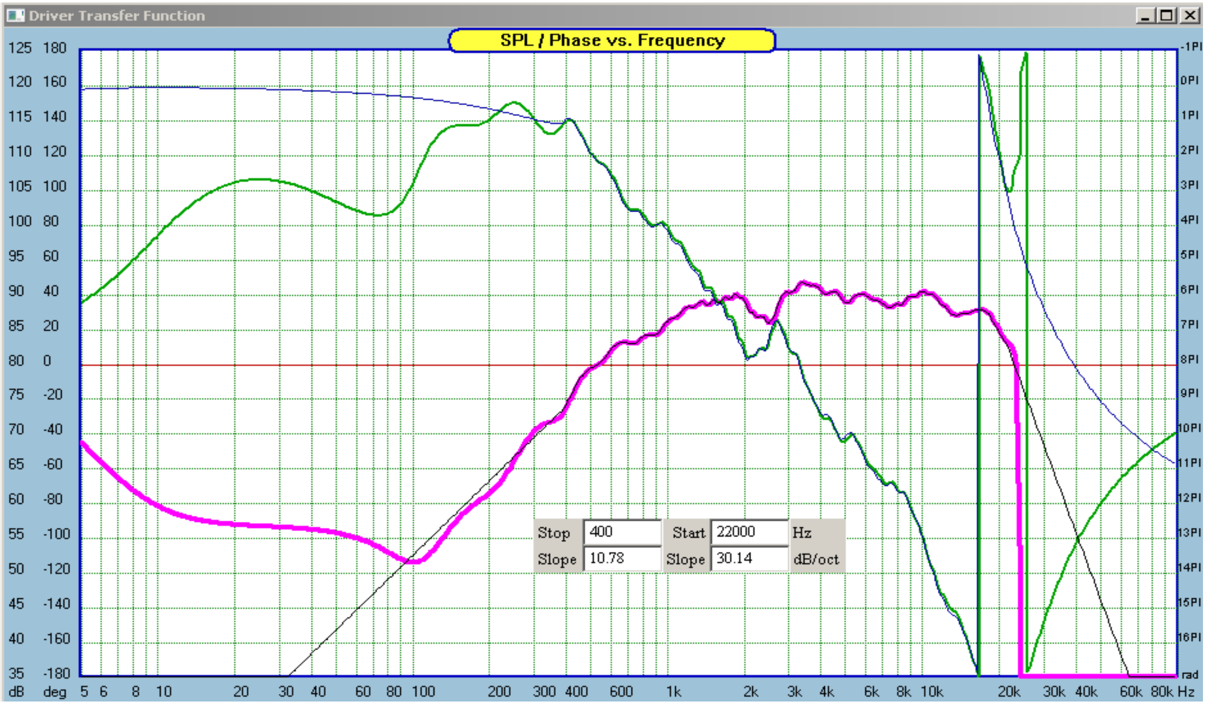


Figure 16.329. Phase response from constrained IHBT method

Lastly, the second woofer frequency response is shown below. Examining the measured SPL, it would be prudent to assume, that break-up region would start above 6kHz. Therefore the highest attachment point would be 6kHz.

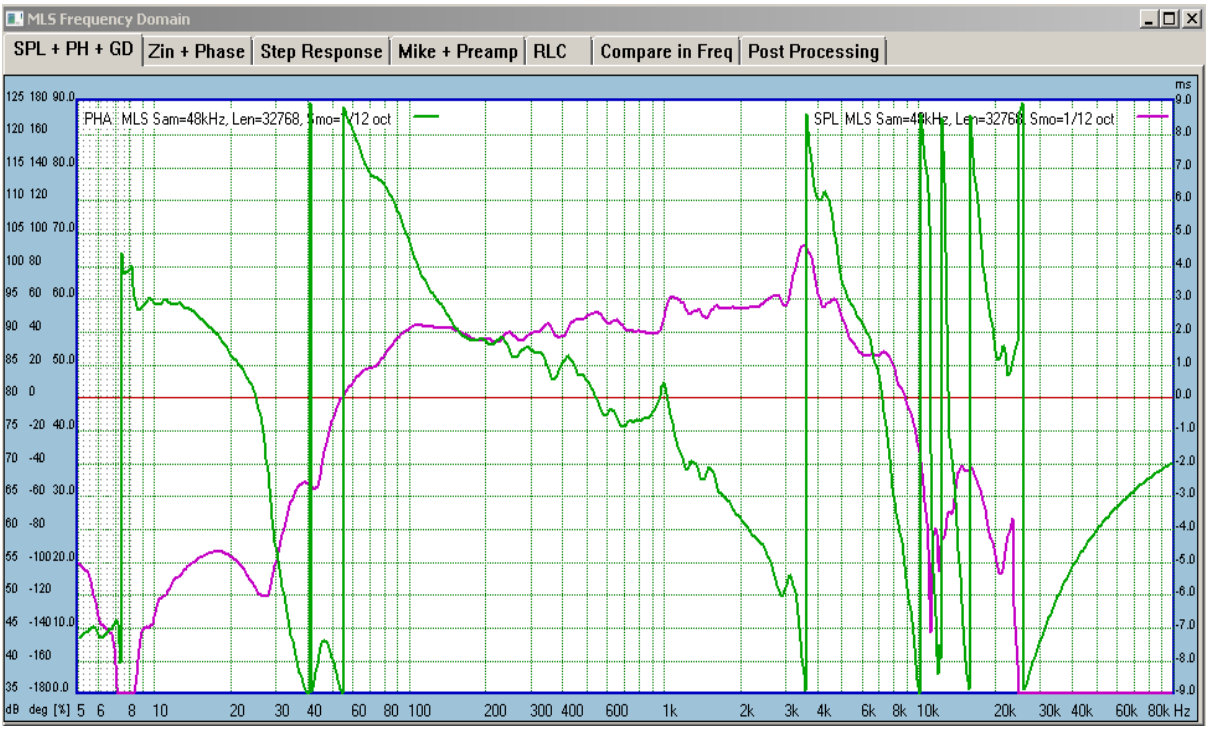


Figure 16.330. Example of SPL/Phase response of a woofer driver.

		Driver's Name		IMP_RES_2.res				
Start Bin =	74	Smooth =	1/12 dB/oct		Window =	120ms		
Start HBT	50Hz	18.5dB	5000	40dB				
						After HBT		Normalized
HBT LP Start	HBT HP Start	Bin	Delay	LP Order	HP Order	Error	Bins	Error
5000	50	78	-0.012	24.87	15.74	2836.715	349	8.12812
6000	50	79	0	13.67	15.76	2939.151	362	8.1192
7000	50	79	-0.002	16.47	15.76	3004.126	374	8.03242
8000	50	78	-0.01	31.85	15.76	2947.261	384	7.67516
9000	50	77	0.004	35.21	15.73	3089.631	393	7.86166
9500	50	77	-0.002	38.34	15.72	3174.942	397	7.99734
10000	50	76	0.008	44.24	15.7	3443.358	401	8.58693

Attachment-dependant tabulated results show the Normalized Error is very close for all attachment points. It would be very challenging indeed to discriminate between those by visually inspecting phase matches.

The 6kHz attachment point was selected for unconstrained HBT method.

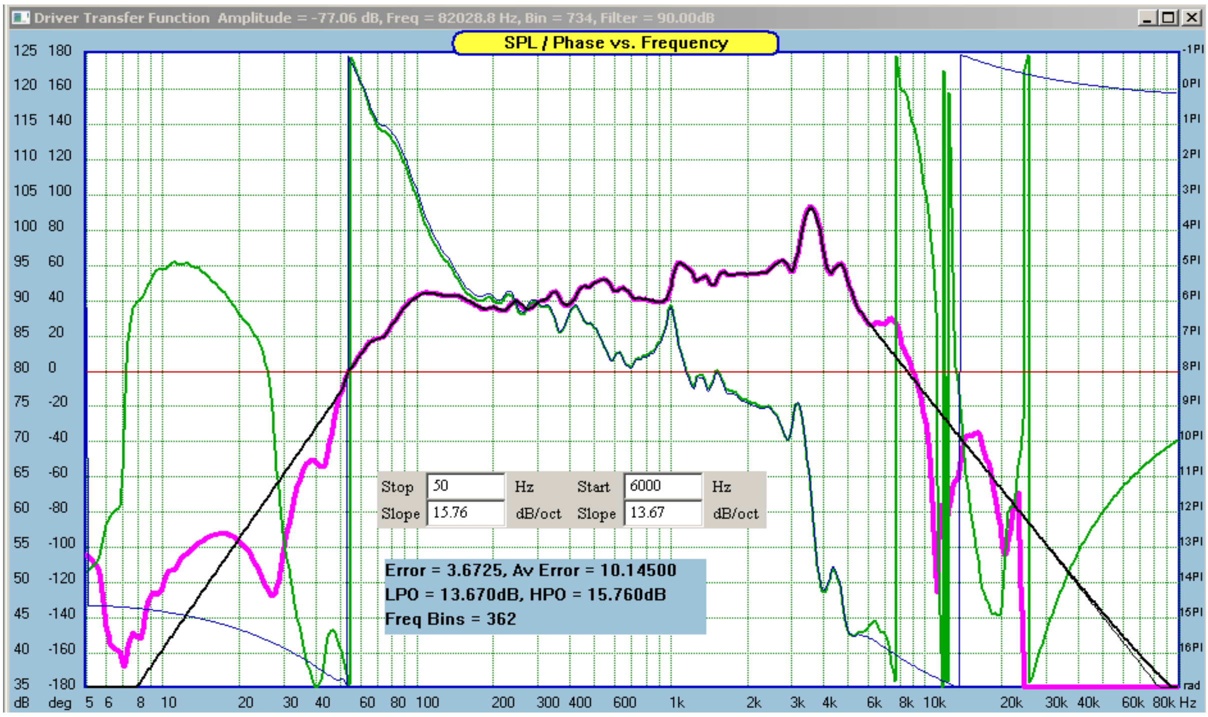


Figure 16.331. Phase response from unconstrained HBT method

The constrained IHBT method resulted in different slopes.

								IHBT	Normalized
IHBT LP Start	IHBT HP Start	Bin	Delay	LP Order	HP Order	Error	Bins	Error	Error
5000	100	76	-0.004	34.02	15.65	3335.05	349	9.44	9.55602
6000	100	77	0	27.18	15.67	3789.58	362	11.66	10.4685
7000	100	78	-0.01	27.39	15.72	3536.02	374	12.51	9.4546
8000	100	77	-0.002	37.36	15.72	3171.76	384	12.99	8.25979
9000	100	77	-0.006	40.04	15.71	3196.92	393	14.31	8.13466
9500	100	77	-0.01	42.4	15.71	3243.13	397	15.85	8.16909
10000	100	76	0.002	47.44	15.68	3501.49	401	21.15	8.7319

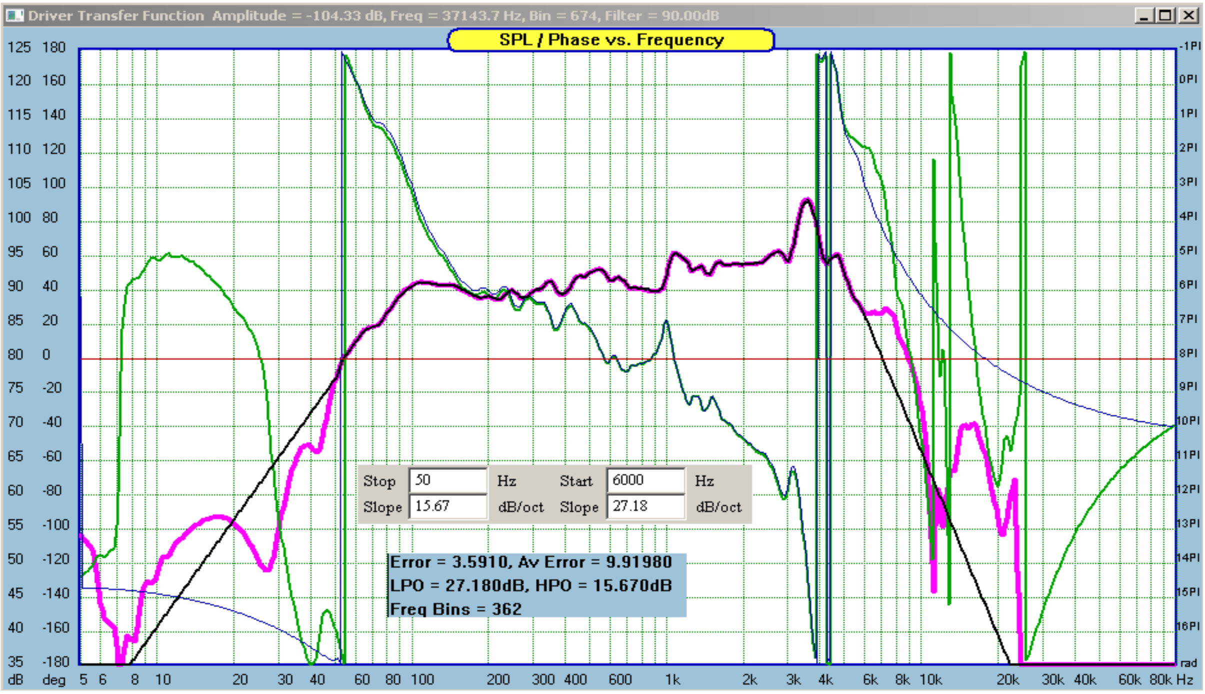


Figure 16.332. Phase response from constrained IHBT method

What is the Constraint?

As observed on several Figures presented so far, manual and automated methods occasionally drift into the "difficult-to-justify" set of parameters, that also produce unusual and unexpectd plots, like Figure 16.321, Figure 16.324 and Figure 16.331.

The algorithms are designed to go and find the minimum error value, and they do it very efficiently, with high degree of accuracy. However, the selection of starting parameters, and the interpretation of the results lies with the human operator. For instance, starting FFT Bin for the optimization process is selected by the operator. Attachment points are selected by the operator, and as we are discussing, the constraint(s) are selected by the operator. There are other starting parameters, for sake of clarity, they will not be discussed here. Some of the reasoning behind selection of the attachment points was presented in this paper and should serve as a guidance for eliminating potential duds.

As it was suggested, that the constraint should be based on SPL response rather than phase response. Frankly speaking, I would not know how to define constraint based on phase response. SPL is much easier to deal with. We all know when the SPL curve does silly things.

Also, if we accept the SPL-based constraint, it <u>must not</u> be a "hard-limit" type of constraint. So, for instance, if one selects 40dB as a constraint for low-pass slope, it does not mean, that low-pass slope will be fixed at -40dB and the rest of the optimization must dance around this limit.

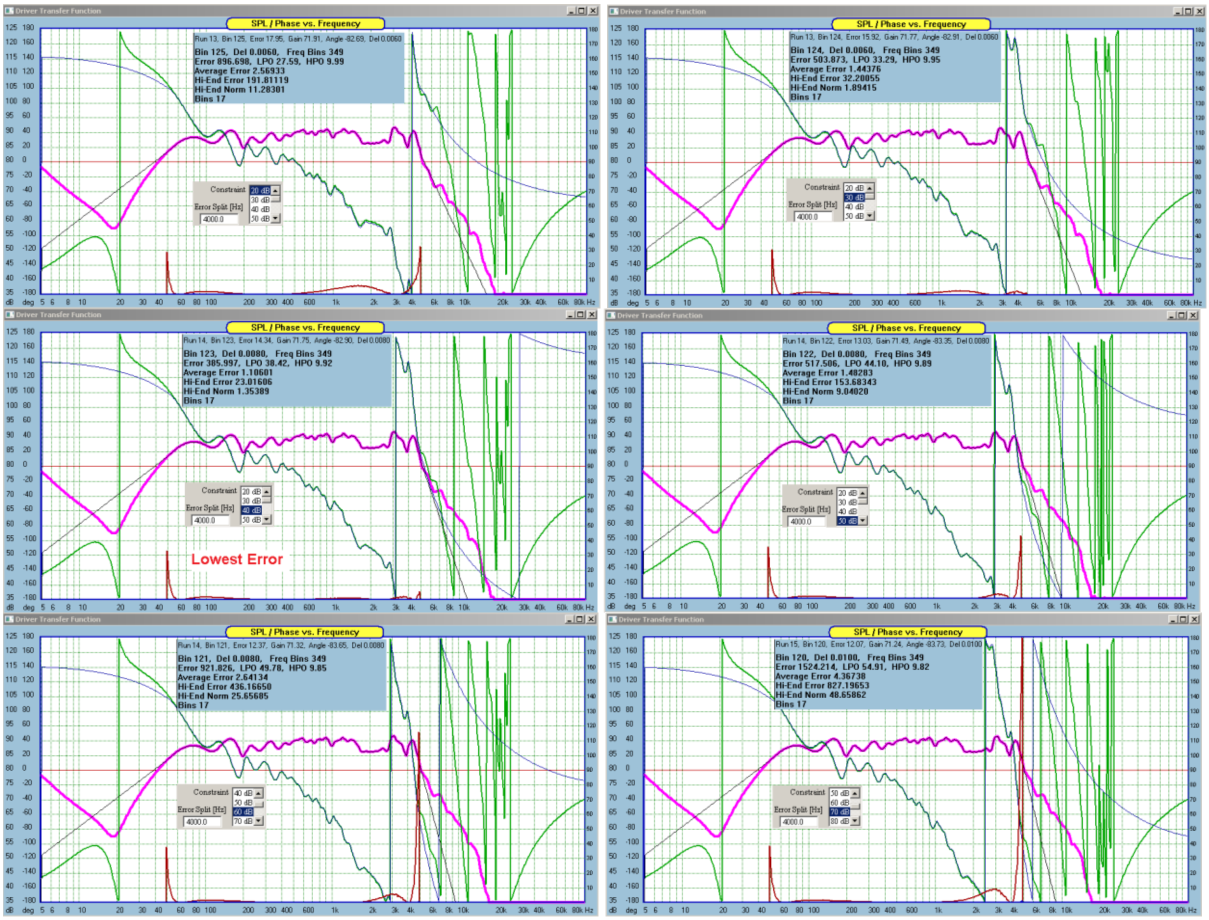
On Figure 16.321, as an example, the unconstrained optimization resulted in phase response corresponding to -8.97dB/oct asymptotic slope. In constrained optimization, the algorithm was asked to show the best phase matches around -30dB/oct asymptotic slope – and the algorithm presented Figure 16.322, with -23.43dB/oct low-pass asymptotic slope.

The unconstraint optimization results shown on Figure 16.331, resulted in the woofer roll-off with berely 2-nd order slope of -13.67dB/oct. While constrained optimization gave more realistic -27.18dB/oct.

Validity of the constraint can be evaluated by selecting different constraint values and running the constrained algorithm for a case presented on Figure 16.326.

		Driver's Name		Rear_Speaker_50cm.res					
IHBT Method									
Start Bin =	117	Smooth =	1/12 dB/oct		Window =	25ms			
Start IHBT	80Hz	Stop IHBT	5000 Hz		HBT	50-5000Hz			
						After HBT		IHBT	Normalized
Constraint [dB]	IHBT HP Start	Bin	Delay	LP Order	HP Order	Error	Bins	Error	Error
20	80	125	0.006	27.59	9.99	896.698	349	17.95	2.56934
30	80	124	0.006	33.29	9.95	503.873	349	15.92	1.44376
40	80	123	0.008	38.42	9.92	385.997	349	14.34	1.10601
50	80	122	0.008	44.1	9.89	517.506	349	13.03	1.48283
60	80	121	0.008	49.78	9.85	921.826	349	12.37	2.64134
70	80	120	0.01	54.91	9.82	1524.214	349	12.07	4.36738
80	80	119	0.01	60.58	9.78	2450.726	349	12.28	7.02214
90	80	118	0.01	66.25	9.74	3647.665	349	13	10.4518
100	80	118	-0.01	71.68	9.72	5047.401	349	14.37	14.4625
110	80	117	-0.008	76.79	9.69	6589.273	349	15.5	18.8804
120	80	116	-0.006	81.9	9.65	8346.337	349	17.6	23.915

The low-pass attachment point was selected as 5000Hz, therefore constrained optimization was run within 80Hz-5000Hz for different Constraint values 20dB – 120dB and the results are tabulated above. The Normalized Error shown on the last column, is clearly the lowest for Constraint = 40dB (green font above). The Error Curve (brown curve) progress is shown on Figure 16.333 below.



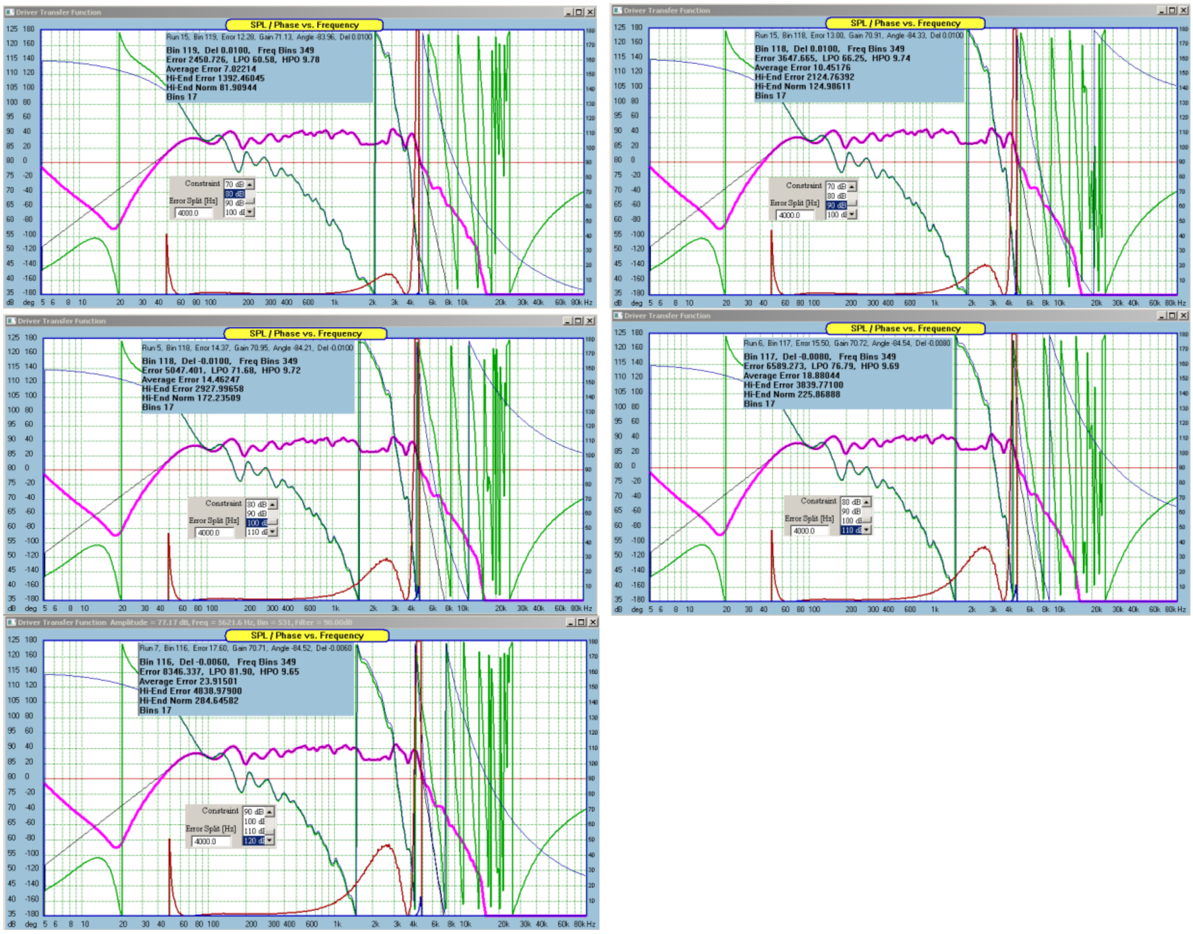


Figure 16.333. Excess phase calculated for different constraint values.

Analyzing Errors

Having the Cumulative Error value and particularly the Normalized Cumulative Error Value assisting in phase extraction decisions is of a great help. But there is more to it.

In the next set of tabulated results, the Cumulative Error was split into Hi-End Error (above 4000Hz) and Low-End Error (below 4000Hz).

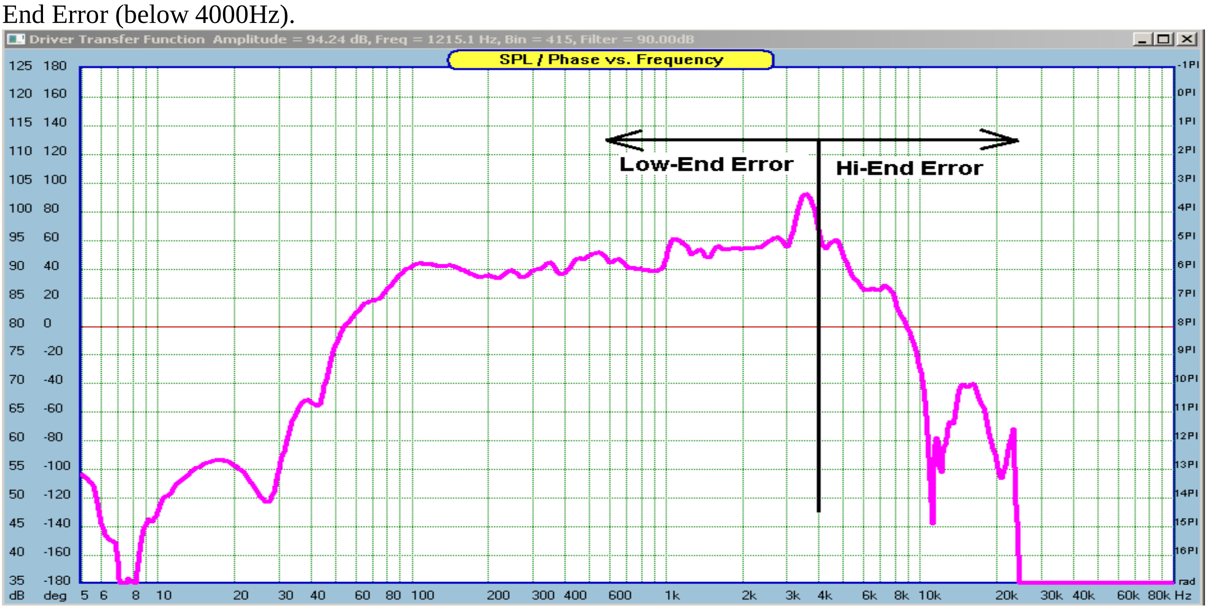


Figure 16.334. Example of splitting Error into two regions.

												4kHz = 505 bin			
Start Bin =	74	Smooth =	1/12 dB/oct		Window =	120ms									
Start HBT	50Hz	18.5dB	5000	40dB								Hi-End			
						After HBT		Normalized		Hi-End	Hi-End	Normalized		Low-End	Normalized
HBT LP Start	HBT HP Start	Bin	Delay	LP Order	HP Order	Error	Bins	Error		Error	Bins	Error		Error	Low-End Error
5000	50	78	-0.012	24.87	15.74	2836.715	349	8.12812		5.48422	17	0.3226		2831.2308	8.527804
6000	50	79	0	13.67	15.76	2939.151	362	8.1192		57.2272	30	1.90757		2881.9239	8.680494
7000	50	79	-0.002	16.47	15.76	3004.126	374	8.03242		86.8848	42	2.06868		2917.2412	8.786871
8000	50	78	-0.01	31.85	15.76	2947.261	384	7.67516		59.4415	52	1.14311		2887.8195	8.698251
9000	50	77	0.004	35.21	15.73	3089.631	393	7.86166		128.042	61	2.09906		2961.5886	8.920448
9500	50	77	-0.002	38.34	15.72	3174.942	397	7.99734		216.979	65	3.33814		2957.9632	8.909528
10000	50	76	0.008	44.24	15.7	3443.358	401	8.58693		408.886	69	5.92589		3034.4716	9.139975
											Growth	18.3692		Growth	1.071785

Probably the most interesting aspect of this analysis is the growth of Hi-End Error and Low-End Error calculated for different low-pass attachment points.

The growth of Normalized Low-End Error is only **1.0717 times** for attachments 5kHz-10kHz
The growth of Normalized High-End Error is a wooping **18.36 times** for attachments 5kHz-10kHz. It is important to calculate Normalized values, because they account for increased number of frequency bins at high-end of the bandwidth.

It is evident, that almost all the growth in Error values comes from drifting into the break-up region. On the next Figure 16.335, the plots also incorporate the Error Value (thick brown line) versus frequency plots. The attachemnt points start at 5kHz and end at 10kHz.

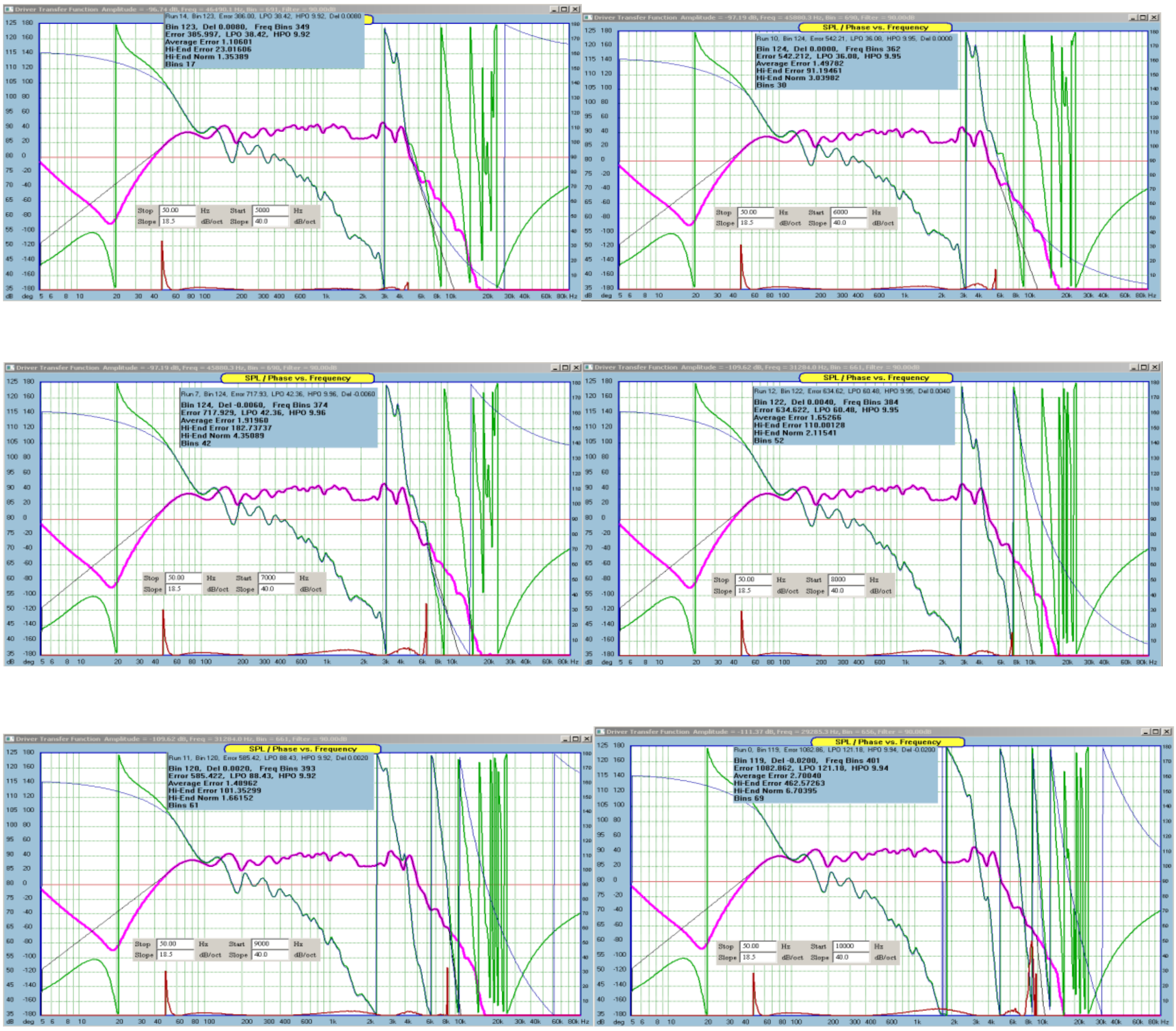
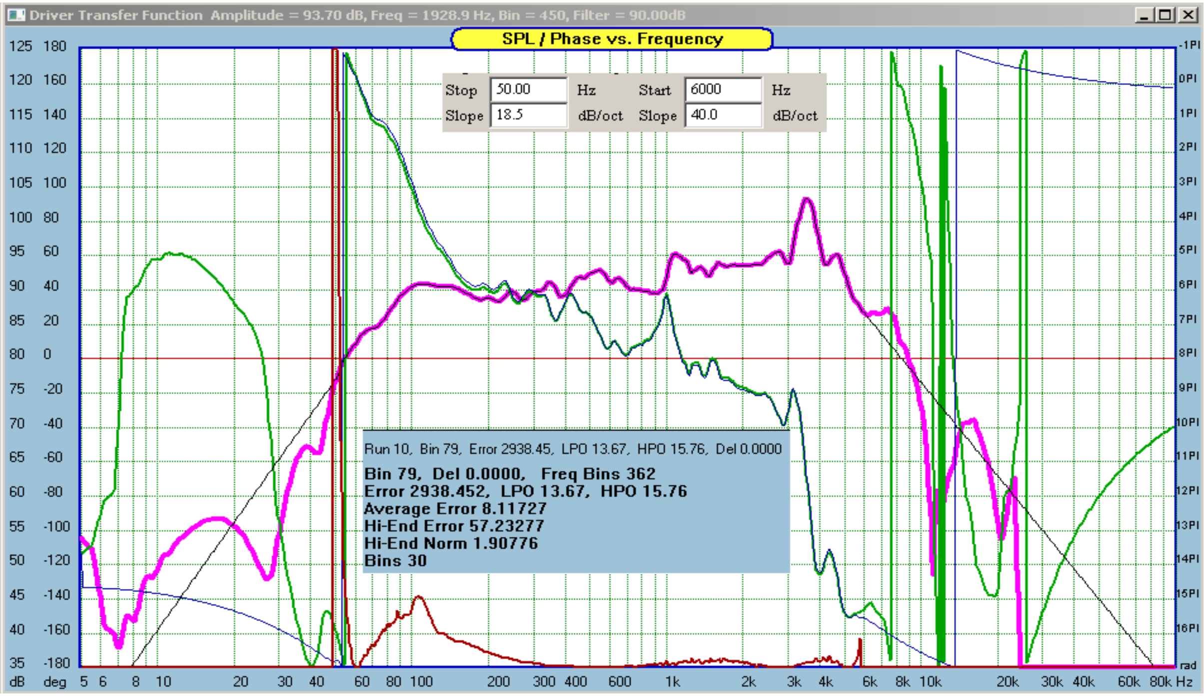


Figure 16.335. Error split into two regions.

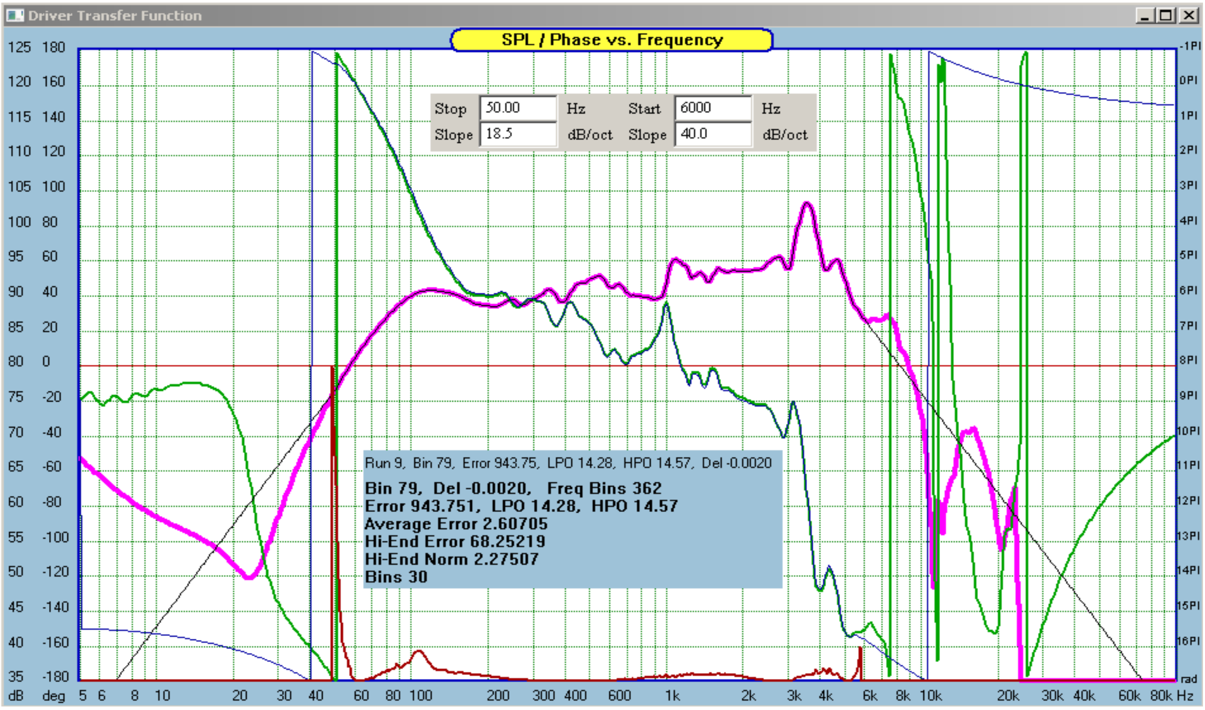
There is very little change in Error Value in the lower frequency range. All the growth comes from moving the attachment point into the break-up region.

The Error Value in the low-frequency region will be affected by Windowing and Smoothing parameters. In this case, the effect is the opposite, and the Error does not change very much at the high end, but (as in the example below), the Error is 3 times lower at low end of the frequency range for narrower FFT Window of 30ms.

FFT Window = 120ms, Smoothing = 1/12dB/oct



FFT Window = 30ms, Smoothing = 1/12dB/oct



Figore 16.337. Low-frequency Error depends on Windowing and Smoothing

Finally, we examine one more tweeter driver. This one has been sampled at 96kHz, using tabulated results for different attachment point, shown below.

		Driver's Name		Raw_Tweeter_0.0deg_100cm.res					
Start Bin	323	Smooth=	1/48dB/oct		Window =	5ms	Time Step =		0.001ms
Start HBT	400	12	40000	40dB					
								Normalized	
HBT LP Start	HBT HP Start	Bin	Delay	LP Order	HO Order	Error	Bins	Error	
38000	400	327	-0.002	50.93	12.84	500.299	345	1.45014	
39000	400	327	-0.003	53.84	12.83	535.64	347	1.54363	
40000	400	327	-0.004	56.95	12.82	539.388	349	1.54552	
41000	400	326	0.005	61.47	12.82	507.413	351	1.44562	
42000	400	326	0.005	67.67	12.81	455.586	353	1.29061	
43000	400	326	0	76.48	12.8	418.701	355	1.17944	
44000	400	326	-0.001	79.88	12.8	432.41	356	1.21463	
45000	400	326	-0.004	89.37	12.8	559.564	358	1.56303	
46000	400	326	-0.004	91.45	12.8	1076.5	360	2.99028	
47000	400	326	-0.004	92.44	12.83	1603.311	361	4.4413	

The SPL and phase response corresponding to the minimum Normalized Error of 1.17944. The final low-pass SPL slope is presented as black thick line of -76dB/oct roll-off.

It seems to be quite fast roll-off, and the combined SPL response looks unnatural. It would be very challenging to visually discriminate for the best phase match between 38kHz right up to 45kHz attachment points. The errors are very close to each other indeed.

Even steeper roll-offs are require to satisfy minimum errors at 44kHz – 47kHz. At 47kHz attachment point the roll-off is -92.44dB/oct.

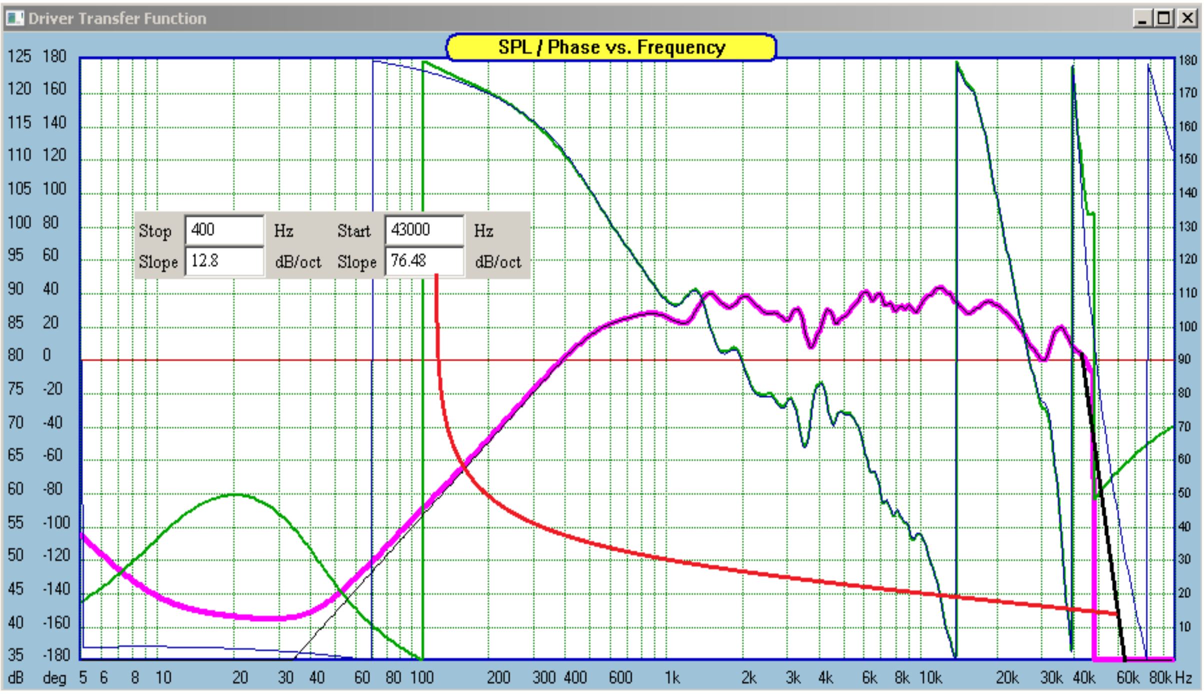


Figure 16.338. High-frequency roll-off seems to steep.

The phase responses for 40kHz and 43kHz attachment points using Un-Constrained HBT method are plotted below.

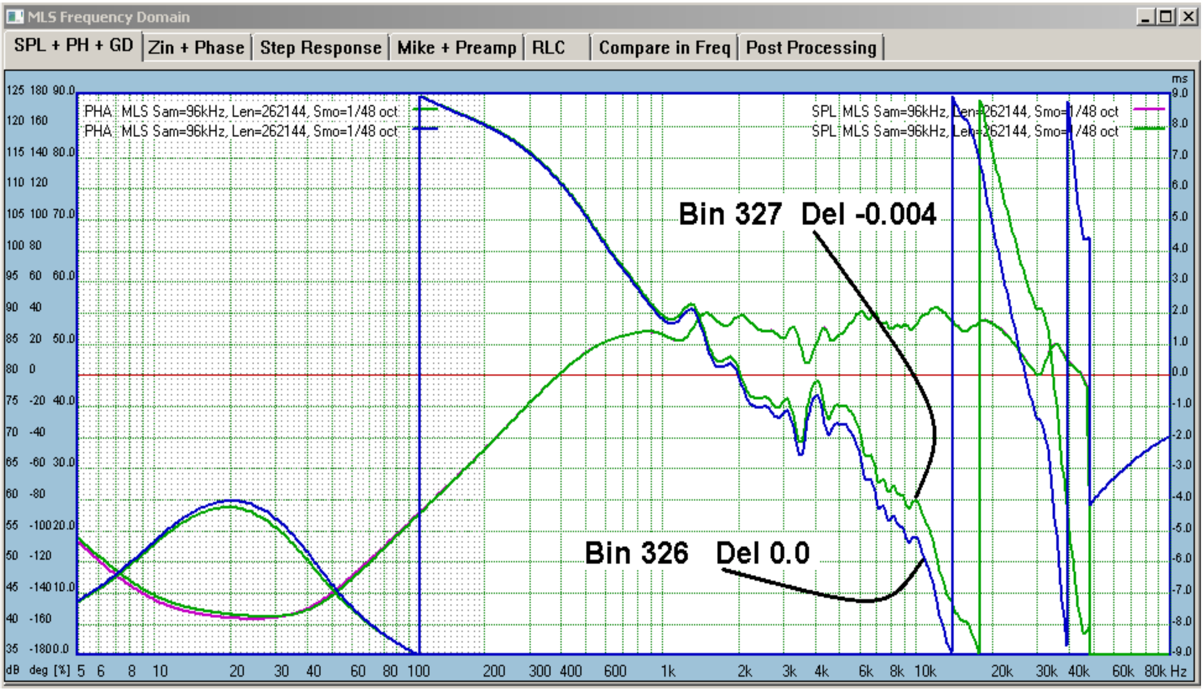


Figure 16.339. Phase responses for 40kHz and 43kHz attachment

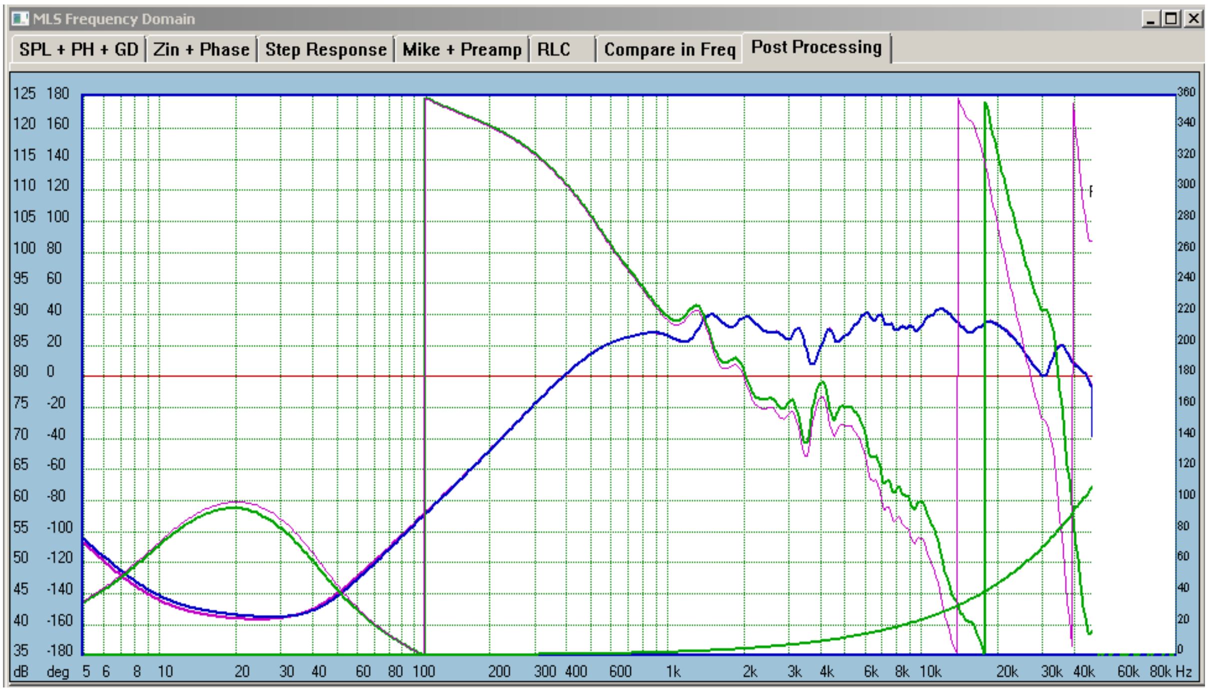


Figure 16.340. It is observable, that the phase difference is 48 deg at 20kHz (green line).

Similarly, the same tweeter was examined using Constrained IHBT Method. Two full runs were conducted. One for 40kHz attachment point and one for 43kHz attachment point

Run 1: From 400Hz To 40000Hz

```
B 327, Del 0.003, E 1434.30, Con 40, NE 4.1097, LP 41.94, HP 12.91, E-LP -1.94
B 327, Del 0.001, E 983.08, Con 45, NE 2.8168, LP 46.23, HP 12.88, E-LP -1.23
B 327, Del 0.000, E 816.17, Con 50, NE 2.3386, LP 48.38, HP 12.87, E-LP 1.62
B 327, Del -0.002, E 599.76, Con 55, NE 1.7185, LP 52.66, HP 12.85, E-LP 2.34
B 327, Del -0.003, E 550.19, Con 60, NE 1.5765, LP 54.82, HP 12.84, E-LP 5.18
B 327, Del -0.004, E 539.44, Con 65, NE 1.5457, LP 56.95, HP 12.82, E-LP 8.05
B 326, Del 0.005, E 616.70, Con 70, NE 1.7671, LP 59.94, HP 12.80, E-LP 10.06
B 326, Del 0.004, E 706.04, Con 75, NE 2.0230, LP 62.08, HP 12.78, E-LP 12.92
B 326, Del 0.002, E 1001.26, Con 80, NE 2.8689, LP 66.36, HP 12.76, E-LP 13.64
B 326, Del 0.001, E 1207.32, Con 85, NE 3.4594, LP 68.51, HP 12.76, E-LP 16.49
B 326, Del 0.000, E 1452.35, Con 90, NE 4.1614, LP 70.66, HP 12.75, E-LP 19.34
B 326, Del -0.001, E 1736.35, Con 95, NE 4.9752, LP 72.79, HP 12.74, E-LP 22.21
B 326, Del -0.003, E 2415.48, Con 100, NE 6.9211, LP 77.08, HP 12.71, E-LP 22.92
B 326, Del -0.004, E 2814.04, Con 105, NE 8.0631, LP 79.22, HP 12.71, E-LP 25.78
B 325, Del 0.005, E 3455.32, Con 110, NE 9.9006, LP 82.26, HP 12.68, E-LP 27.74
```

Run 2: From 400Hz To 43000Hz

```
B 327, Del 0.000, E 2797.17, Con 40, NE 7.8794, LP 52.28, HP 12.93, E-LP -12.28
B 327, Del -0.001, E 2356.65, Con 45, NE 6.6385, LP 54.59, HP 12.92, E-LP -9.59
B 327, Del -0.002, E 1961.28, Con 50, NE 5.5247, LP 56.91, HP 12.90, E-LP -6.91
B 327, Del -0.003, E 1611.23, Con 55, NE 4.5387, LP 59.24, HP 12.90, E-LP -4.24
B 327, Del -0.004, E 1306.72, Con 60, NE 3.6809, LP 61.55, HP 12.89, E-LP -1.55
B 326, Del 0.005, E 995.68, Con 65, NE 2.8047, LP 64.89, HP 12.87, E-LP 0.11
B 326, Del 0.003, E 629.74, Con 70, NE 1.7739, LP 69.53, HP 12.85, E-LP 0.47
B 326, Del 0.002, E 513.85, Con 75, NE 1.4475, LP 71.84, HP 12.83, E-LP 3.16
B 326, Del 0.001, E 443.66, Con 80, NE 1.2497, LP 74.15, HP 12.82, E-LP 5.85
B 326, Del 0.000, E 418.70, Con 85, NE 1.1794, LP 76.48, HP 12.81, E-LP 8.52
B 326, Del -0.001, E 438.50, Con 90, NE 1.2352, LP 78.79, HP 12.79, E-LP 11.21
B 326, Del -0.003, E 612.59, Con 95, NE 1.7256, LP 83.42, HP 12.77, E-LP 11.58
B 326, Del -0.004, E 767.18, Con 100, NE 2.1611, LP 85.73, HP 12.77, E-LP 14.27
B 326, Del -0.005, E 966.67, Con 105, NE 2.7230, LP 88.05, HP 12.75, E-LP 16.95
B 325, Del 0.004, E 1314.76, Con 110, NE 3.7036, LP 91.34, HP 12.74, E-LP 18.66
```

The SPL constraint (Con XX above) for low-pass slope was varied from 40dB to 110dB and IHBT method was run for all these values. Resulting FFT Bin values (B XXX above), Error Values (E XXX.X above), Normalized Error (NE X.XXXX above) Low-Pass slope (LP XX.XX above) and the difference between Constraint – Low-Pass slope (E-LP X.XX) were tabulated.

It is observable, that E-LP parameter can be negative, zero (or close to zero) or positive. **When the E-LP = 0, the constraint is not active, and the process can be considered unconstrained**.

For E-LP = 0.11 (close to zero), the algorithm recommends 64dB/oct Low-Pass slope.

The recommended excess phase values were plotted for both attachment points 40kHz and 43kHz, and compared – see plots below.

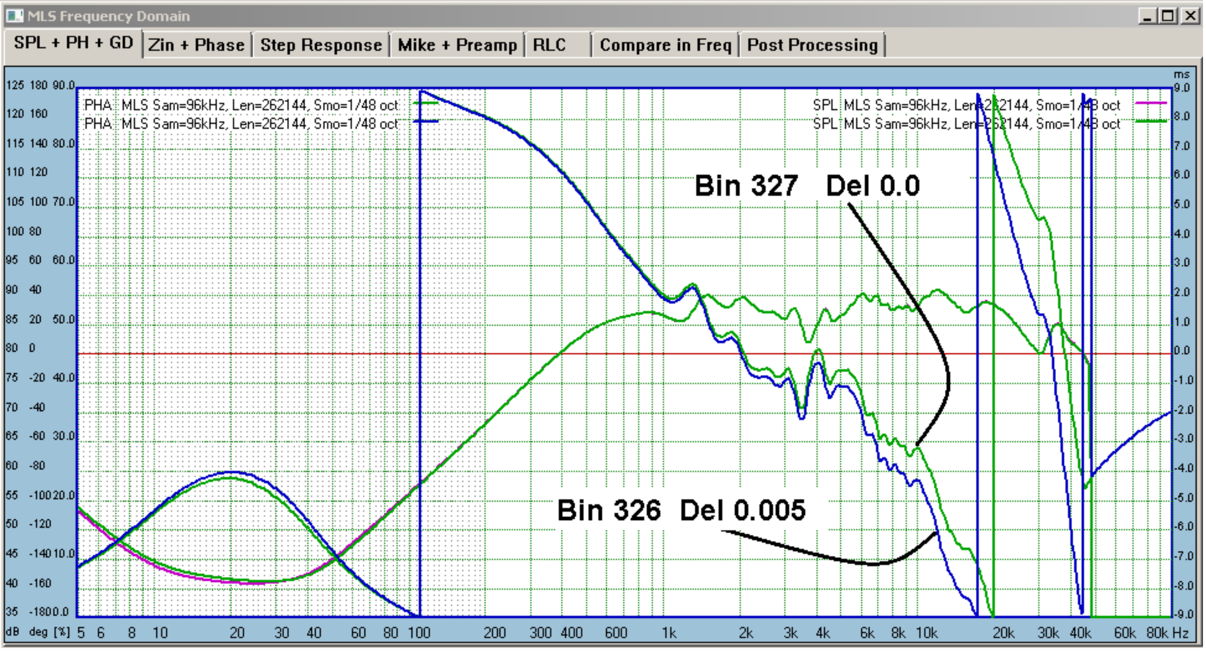


Figure 16.341. 40kHz and 43kHz Constrained IHBT method

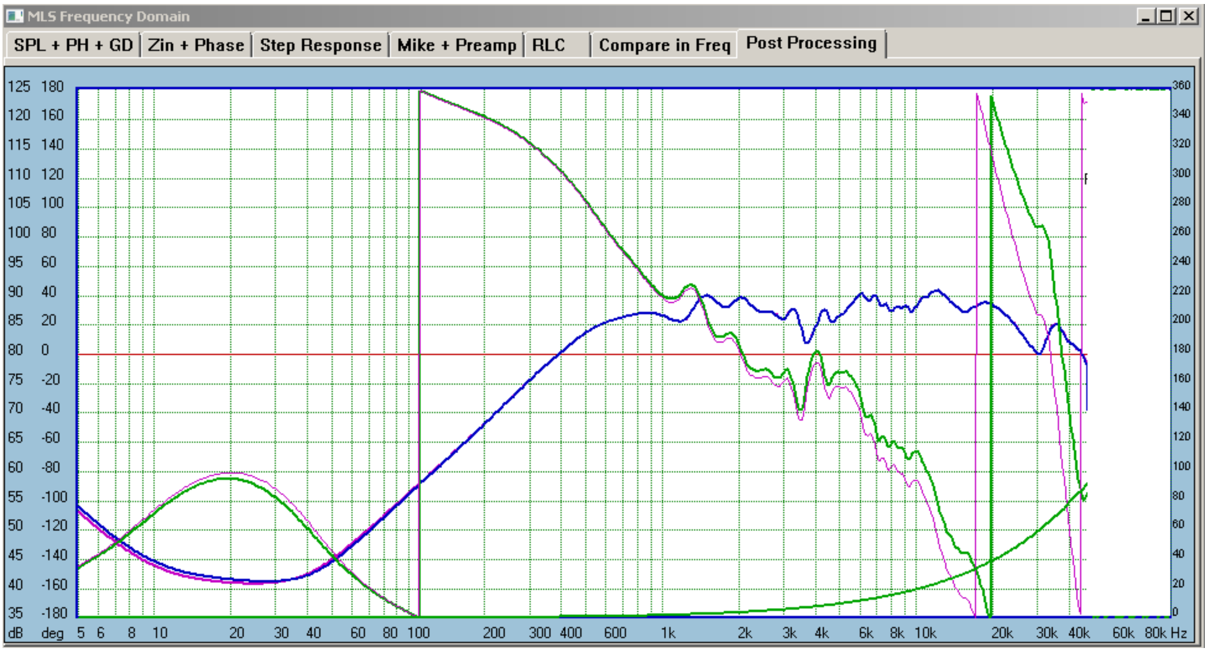


Figure 16.342. Constrained IHBT Phase Difference 38 deg at 20kHz (green line).

It is observable, that Constrained HBT method offered reduction in excess phase difference from 48 deg to 38deg at 20kHz, which is by 20.8%.

This is not a large value, but combined with the recommended slope of 64dB/oct (vs 76dB/oct) seems to be shifting the excess phase result in the right direction.

One final interesting observation: the averaged value of 4 slopes for attachment poins between 40kHz, 41kHz, 42 kHz and 43kHz is equal to 65.64dB/oct. It may not be a bad idea, to select 41.5kHz attachment point and 65dB low-pass slope. Both methods seem to be pointing to the same conclusion.

Once the measurement process using MLS or ESS methods is completed, you are presented with SPL and Phase responses. Next comes the all-important question: “And now what?”.

Minimum-Phase extraction process can be rather complicated process. In order to lessen the burden, a selection of tools and methods has been devised, encompasing (1) inclusion of all measurement processes and measurement options, (2) automation, (3) various numerical error presentations, (4) Error vs. frequency display curve and (5) common senese analysis of all these results. It is advisable to use as many tools as there are available to estimate the elusive minimum-phase response of a measured driver.

Most often, the smallest error will be the best indication of the phase correctness. Other times, bizzare SPL curve will eliminate some results, and what’s left would be the correct phase outcome. Finally, analysis of the measured SPL/Phase curve will offer some clues as to what the next step should be in selecting automation options, particularly when strategically selecting the attachment points. Some more insight can be provided by the IHBT method here.

When analysing the results, it would be prudent, to take into account sensitivities of the whole phase extraction to changes in parameters of measurement process. For instance, it would be incorrect to compare phase errors calculated for different windowing or smoothing parameters. The two Windowing example figures presented before show the Average Error = 2.60705 for 30ms FFT Window, as opposed to Average Error = 8.11727 for 120ms FFT Windows. Parameters of the measurement process need to be selected based on sound measurement practices, and kept constant for all phase extraction activities.

Cumulative Error is a good indication of where the things are going globally. Normalized Error helps to determine if extending the bandwidth via moving the attachment points causes the error to grow unusually large. For instance, if the phase difference between measured and HBT-drived phase is a steady 1 degree per frequency point, the Cumulative Error over 1 data point will be equal to 1. Cumulative Error for extended bandwith of 10 data points will be equal to 10. But Normalized error will be still equal to 10/10 = 1. So, it will be the Normalized Error, that will alert you to unusually large phase gaps. Examining the Error vs. frequency plot is also beneficial.

Normalized Error vs. Constraint tabulated results and Normalized Error vs Attachment Point tabulated results would ideally reconcile with the same location of the FFT window and delay, and therefore the same excess phase.

The HBT Method and IHBT Method described above, work essentially off the same principles, therefore, they produce the same results. The methods were designed to compliment each other, and act as a consistency check for each other. Any differences could be attributed to the finite time-step resolution in algorithmic implementation and are negligible. The processes allow the user to extract minimum-phase response with +/-1usec accuracy when used with 96kHz sampling.