# Automated Method For Minimum-Phase Extraction Using HBT

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## Introduction

When the HBT Method was first introduced, about 20 years ago, there were attempts to implement the method for the purpose of extracting minimum-phase response from a measured SPL response. The HBT Method is based on Dr Bode's integral formulation.

To solve Bode's integral as intended, one needs the knowledge of the function's behaviour at infinity. This translates into simpler language as: need to know the asymptotic behaviour as the function approaches infinity – the asymptotic slopes.

This implies, that the intended usage of the integral is to: (1) provide asymptotic slopes on the low-frequency side and high-frequency side and (2) then calculate minimum-phase response from known SPL data. The process will yield the correct result, as long as the whole function is of "minimum-phase" type.

Then, using the phase result as a template, one can adjust the excess phase in the measured result to match the calculated minimum-phase – thus obtain the desired SPL/Phase measurement with minimum-phase. Simple, if the slopes are known.

In practical terms, this process involved repeated adjustments of 4 parameters driving the HBT and also manipulating the delay time (excess phase) introduced inevitably in measurement process. The idea was that at some point during the adjustments, there will be nearly exact match between the measured phase and HBT-generated phase. At this point, the HBT adjustment parameters and the excess phase would define the minimum-phase match.

## The Manual Method

If you wish to "optimize-by-hand" the process of finding the minimum-phase, then you are being asked to juggle 5 arbitrarily selected parameters: two attachment points, two asymptotic slopes and one excess phase data. During this process, and for each attempt, the user will have to eyeball two phase responses for the best match, while trying to remember how good was the match for the parameters selected before for other set of parameters. It is difficult to assess if you are moving in the right direction as there is no indication guiding the next step. Number of combinations is staggering, particularly when you start moving around the attachment points.

Ambiguity of the eyeballing, lack of numerical information about the progress, tediousness and length of the process are just some of the drawbacks of the manual process.

Visual inspection can be very difficult. Figure 1 shows two phase responses: (1) 36dB/oct low-pass filter (**blue**) and (2) 30dB/oct low-pass filter with 20usec delay added to it (**green**). It is observable, that up to 10kHz, the two phase responses are nearly identical. They start to diverge rapidly above 10kHz. The operator would easily accept this result as perfect match – if the frequency range was limited to 10kHz. Then, the excess phase would never be determined.

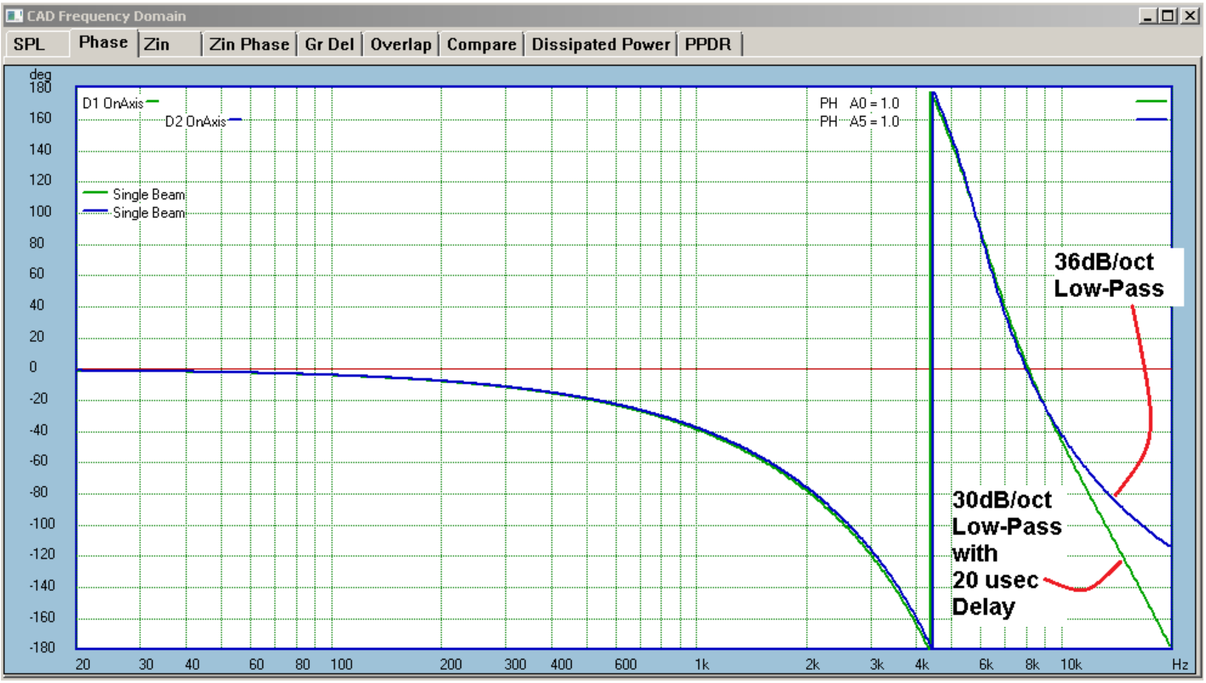


Figure 1. 36dB/oct LP filter (Blue) and 30dB/oct LP filter with 20usec delay.

The phase response of the 6kHz low-pass filer shown on Figure 1, would represent a typical mid-woofer, with frequency bandwidth up to 6 kHz and break-up region above that point. So, it would be a challenge to determine minimum-phase for such driver, given similarity of the phase responses up to 10kHz.

When one relates this issue to loudspeakers in general, it would be prudent to examine the problem of phase matching, over the widest possible frequency range for three reasons, and one opposing.

1. The error between the measured phase and HBT-derived phase will manifest itself more strongly towards the higher frequencies. The error may rapidly increase for wider frequency spans.
2. The high-frequency end, may contain very valid measured data for SPL and phase, so purposefully eliminating this data would lead to diminished confidence in the overall results.
3. When calculating the cumulative error over some frequency range, the error will always be smaller for narrower frequency range. This will leave the operator with the impression, that the set of parameters corresponding the lower error, is the one that should be accepted.
4. On the other hand, attempts to include loudspeaker's break-up region in the process, may back-fire. The break-up region is known as non-minimum-phase region, and it will distort the overall results and accuracy.

## Automated Method

The Automated Method involves one (or more) minimum-seeking algorithms to manipulate available parameters in order to minimize the error between measured phase and the HBT-derived phase.

In short, for each optimization attempt, the whole measurement process is executed by including: (1) selection of windowing parameters and positioning of the FFT window, (2) FFT algorithm, (3) SPL/phase smoothing parameters, (4) Mike Cal file, (5) adding delays, and (6) minimum-phase extraction. Overall, it's not a simple process, even with automation.

The process starts with selecting the low-frequency and high-frequency attachment points (frequencies). Then, for each FFT window position, the measured phase is calculated using FFT algorithm. Next, the optimizing algorithm adjusts the low-end and high-end slopes for the minimum error between HBT-derived phase and measured phase. The error is presented as numerical value, and can be later used to decide which set of HBT parameters and what position of the FFT window and delays delivered the smallest error – meaning, the best match between the measured phase and HBT generated phase. Then another set of attachment points is selected and the process is repeated.

**Example of Tweeter 1**

To illustrate the above concerns, we start with a tweeter driver. The SPL and phase responses are shown on Figure 2. The location of the FFT window was arbitrary, so the presented phase response is not the minimum-phase response.

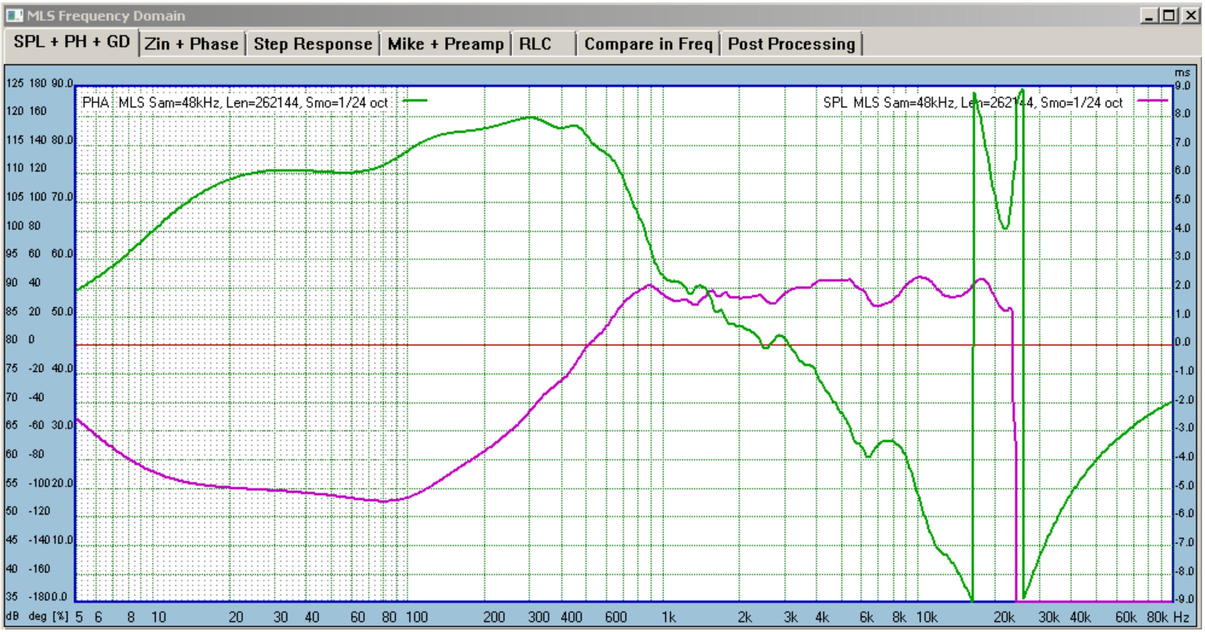


Figure2. SPL/phase example – Tweeter 1 driver.

When different attachment point (frequency) is selected, the resulting phase response will change, therefore, the user is expected to adjust the high-pass or low-pass slopes to compensate for the change in the attachment point location.

Please note, that the excess phase has not changed, so re-adjusting the slopes should only compensate for the change in the attachment point.

This assumption has been tested using the Automated Method.

The results are tabulated below.

| HBT LP Start | HBT HP Start | FFT Bin | Delay ms | LP Order | HP Order | Error |
|---|---|---|---|---|---|---|
| 17000 | 400 | 90 | 0.002 | 8.94 | 11.99 | 330.61 |
| 18000 | 400 | 90 | -0.002 | 14.38 | 12.01 | 366.99 |
| 19000 | 400 | 90 | -0.002 | 14.75 | 11.99 | 399.04 |
| 20000 | 400 | 90 | 0 | 11.93 | 12.01 | 423.03 |
| 21000 | 400 | 90 | 0.002 | 8.97 | 12.01 | 464.91 |
| 22000 | 400 | 90 | 0 | 10.85 | 11.98 | 631.01 |

It is observable, that the algorithm has adjusted the low-pass and high-pass slopes for different attachment points, but is has also suggested different delays (Bin/Delay) – meaning different excess phases for each new attachment point. This will be evident in tabulated results for other drivers.

Recommended phase response for attachment point of 17000Hz is shown on Figure 3 below.

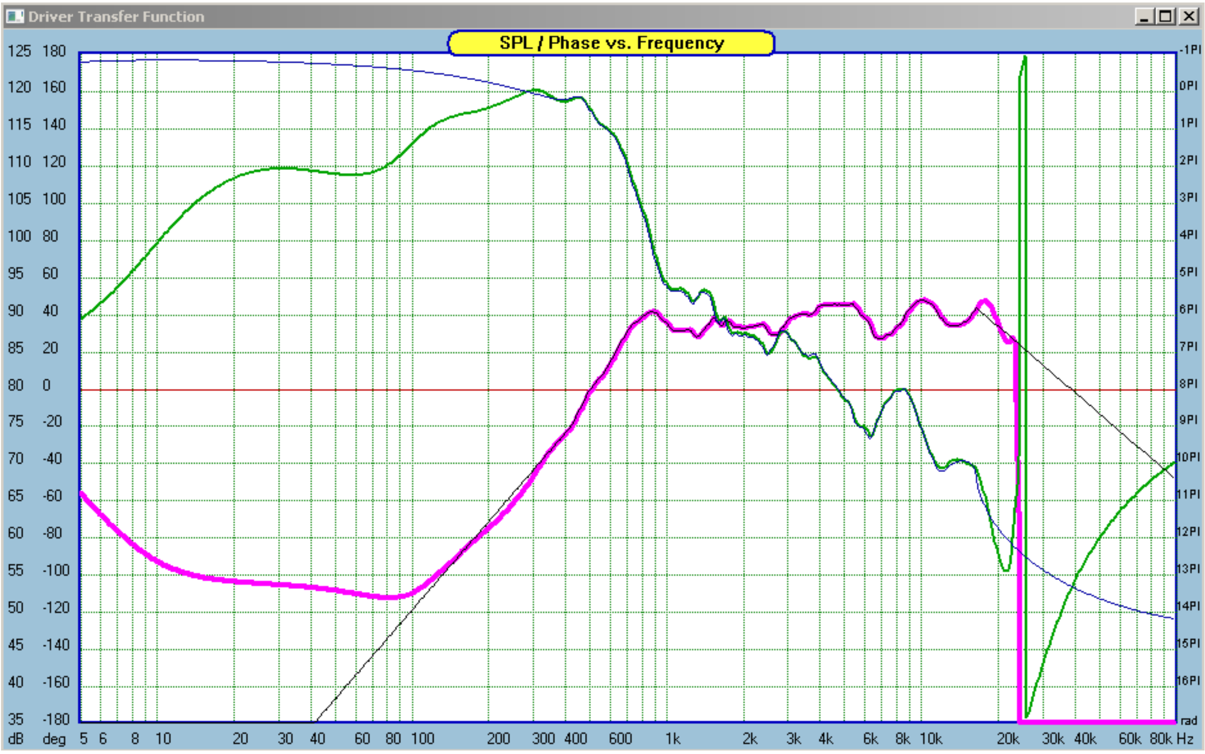

Figure 3. Phase response for the attachment point of 17000Hz.

It is evident on Figure 3, that the attachment point was selected incorrectly, and a 5kHz portion of usable SPL between 17kHz and 22kHz has been neglected in optimizations. The result is unnaturally shallow high-frequency slope of 8.94dB/oct and a-typical phase response. Interestingly, this attachment point resulted in the lowest Error – because of the narrow frequency range selected.

More acceptable phase response is shown on Figure 4 below. It was generated for the following HBT parameters:

| HBT LP Start | HBT HP Start | FFT Bin | Delay ms | LP Order | HP Order |
|---|---|---|---|---|---|
| 22000 | 410 | 89 | 0.006 | 28.42 | 11.96 |

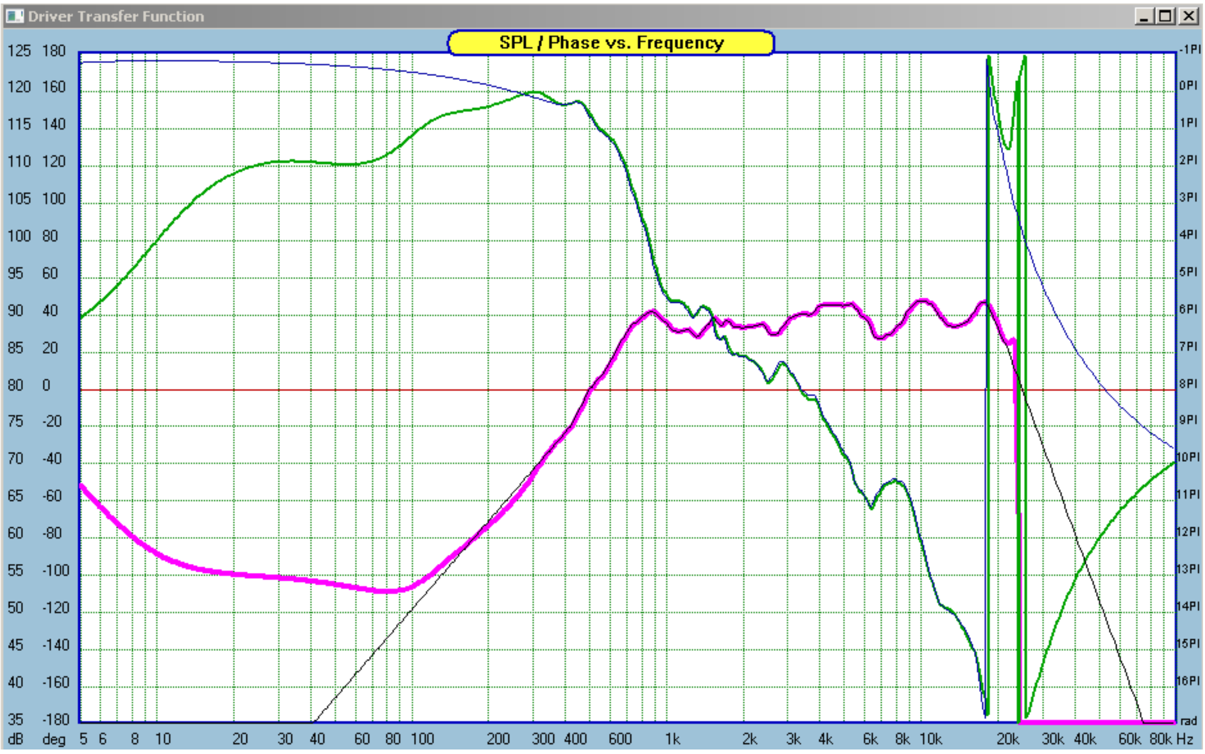


Figure 4. Minimum-phase phase response for Tweeter 1.

### Example of Tweeter 2

Once again, the location of the FFT window was arbitrary, so the presented phase response is not the minimum-phase response. The SPL of Tweeter 2 is shown on Figure 5 below.

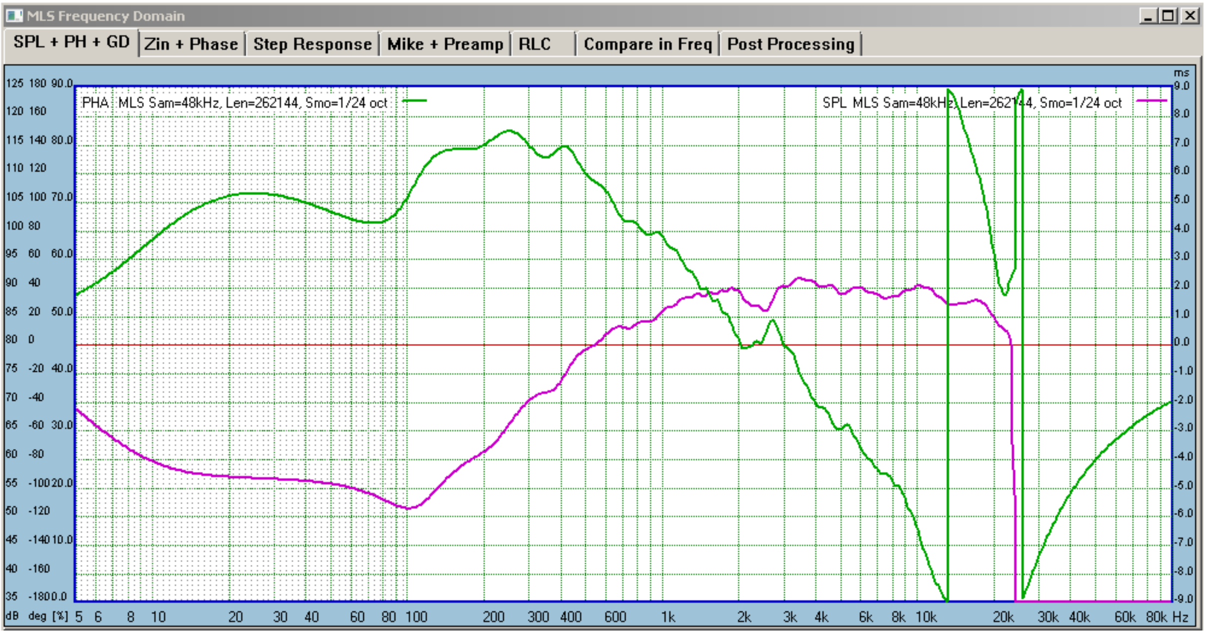


Figure 5. SPL/phase example – Tweeter 2 driver.

Tabulated responses for the Automated Method for different attachment point are shown below.

| HBT LP Start | HBT HP Start | Bin | Delay | LP Order | HP Order | Error |
|---|---|---|---|---|---|---|
| 17000 | 400 | 122 | -0.002 | 15.65 | 11 | 298.76 |
| 18000 | 400 | 122 | -0.004 | 18.76 | 11 | 358.33 |
| 19000 | 400 | 122 | -0.004 | 19.29 | 11.01 | 420.82 |
| 20000 | 400 | 122 | -0.004 | 19.35 | 11.01 | 433.61 |
| 21000 | 400 | 122 | -0.006 | 21.49 | 10.99 | 494.71 |
| 22000 | 400 | 121 | 0.01 | 27.77 | 10.79 | 751.69 |

The problem repeats itself – the attachment point of 17000Hz results in phase response on Figure 6 below.

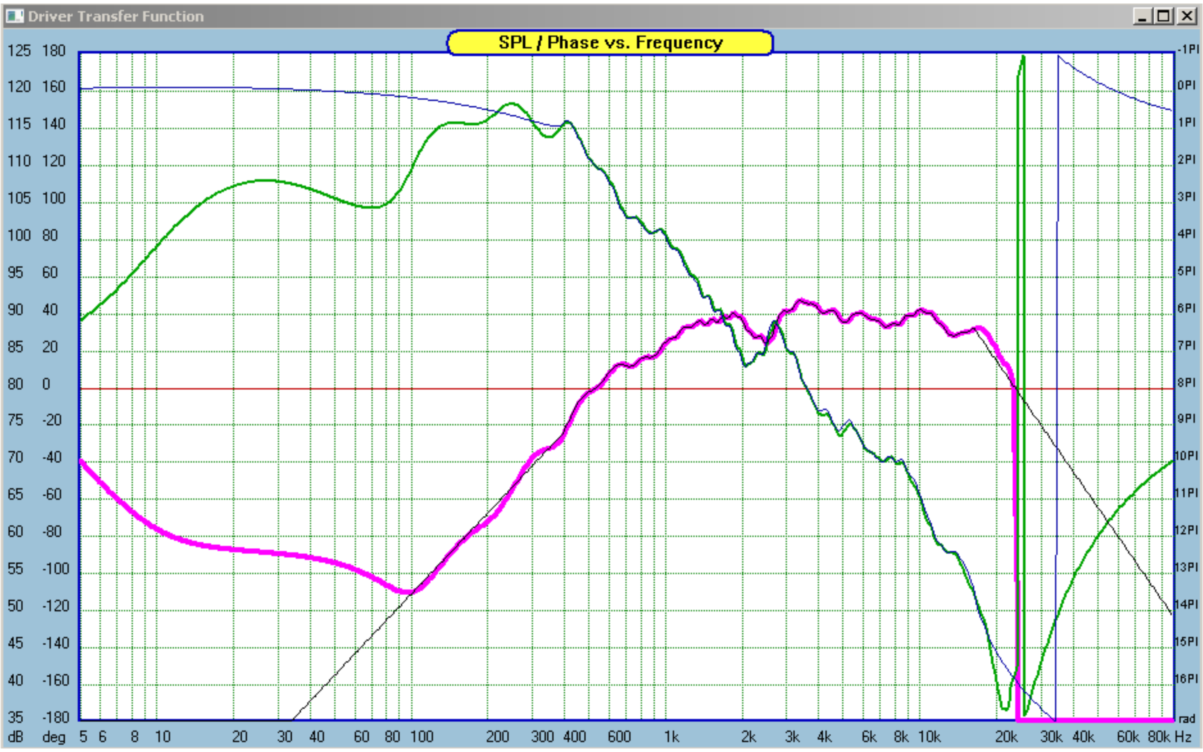


Figure 6. Phase response for the attachment point of 17000Hz.

More acceptable phase response is shown on Figure 7 below. It was generated for the following HBT parameters:

| HBT LP Start | HBT HP Start | Bin | Delay | LP Order | HP Order |
|---|---|---|---|---|---|
| 22000 | 410 | 121 | 0.004 | 34.87 | 10.74 |

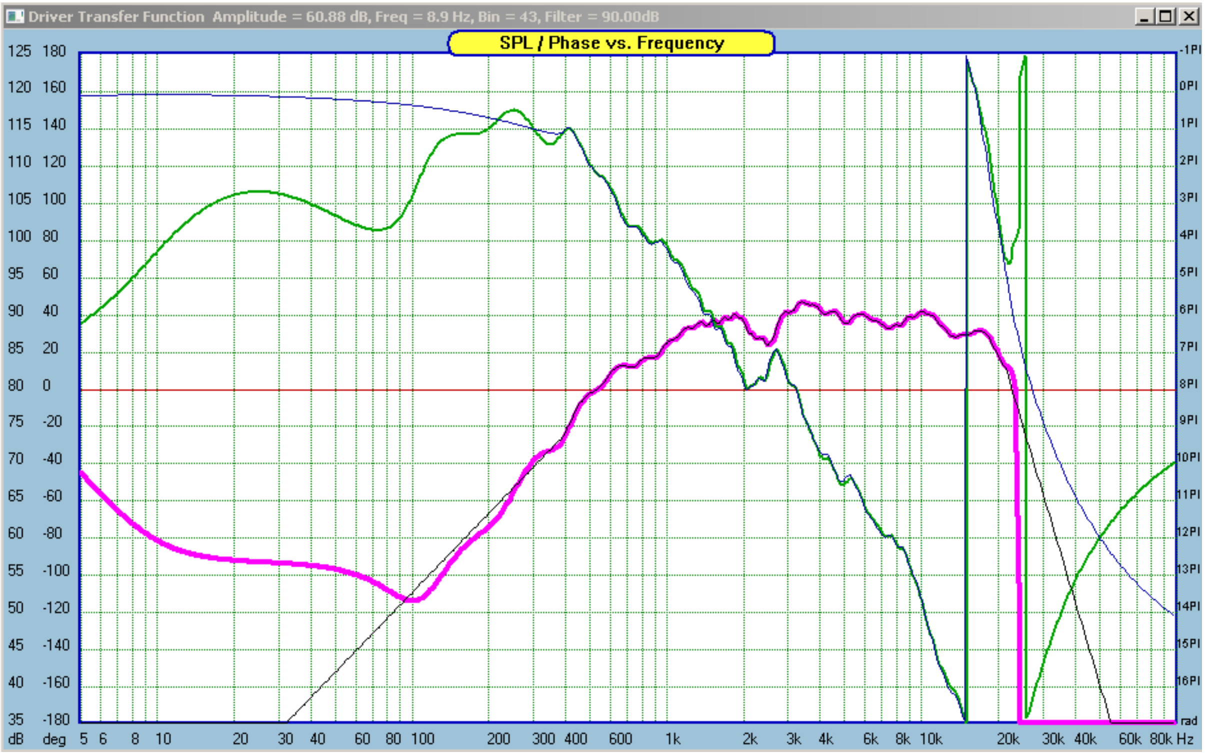

Figure 7. Minimum-phase phase response for Tweeter 2.

### Example of Woofer 1

Once again, the location of the FFT window was arbitrary, so the presented phase response is not a minimum-phase response. The SPL of Woofer 1 is shown on Figure 8 below.

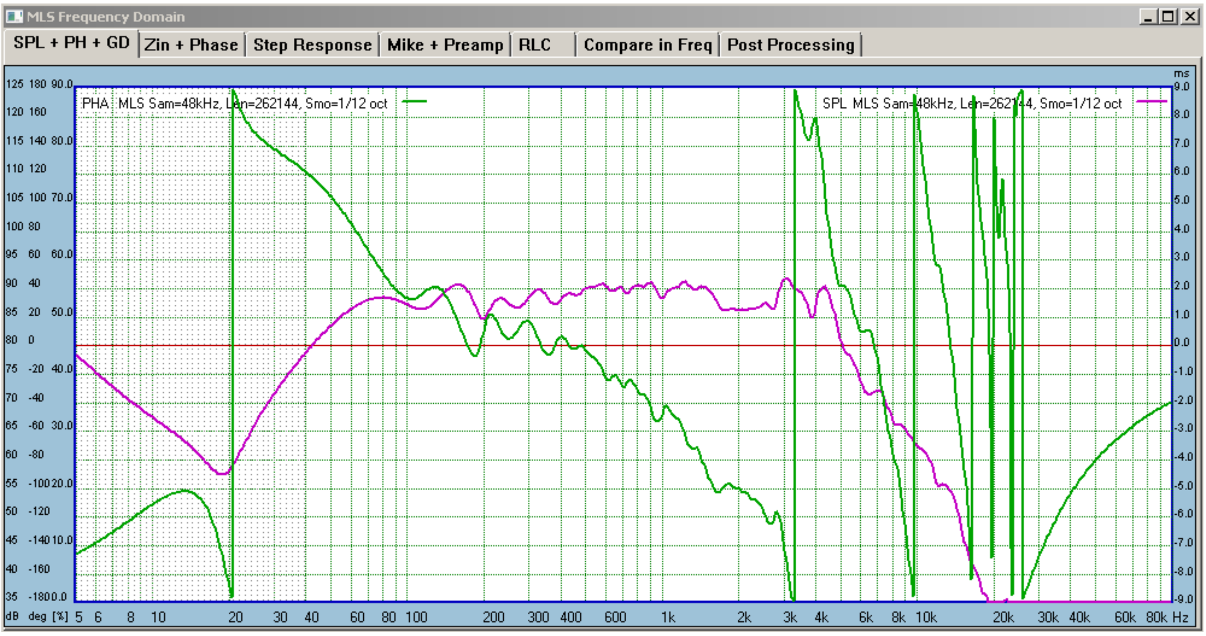

Figure 8. SPL/phase example – Woofer 1 driver.

Tabulated responses for the Automated Method for different attachment point are shown below.

| HBT LP Start | HBT HP Start | Bin | Delay | LP Order | HP Order | Error |
|---|---|---|---|---|---|---|
| 4000 | 50 | 127 | 0.008 | 12.5 | 9.56 | 486.69 |
| 5000 | 50 | 123 | 0.008 | 38.42 | 9.92 | 385.42 |
| 6000 | 50 | 124 | 0 | 36.08 | 9.95 | 542.29 |
| 7000 | 50 | 124 | -0.006 | 42.36 | 9.97 | 718.35 |
| 8000 | 50 | 122 | 0.004 | 60.47 | 9.95 | 635.11 |
| 9000 | 50 | 120 | 0.002 | 88.43 | 9.93 | 586.06 |
| 10000 | 50 | 117 | 0.006 | 129.45 | 9.9 | 770.97 |

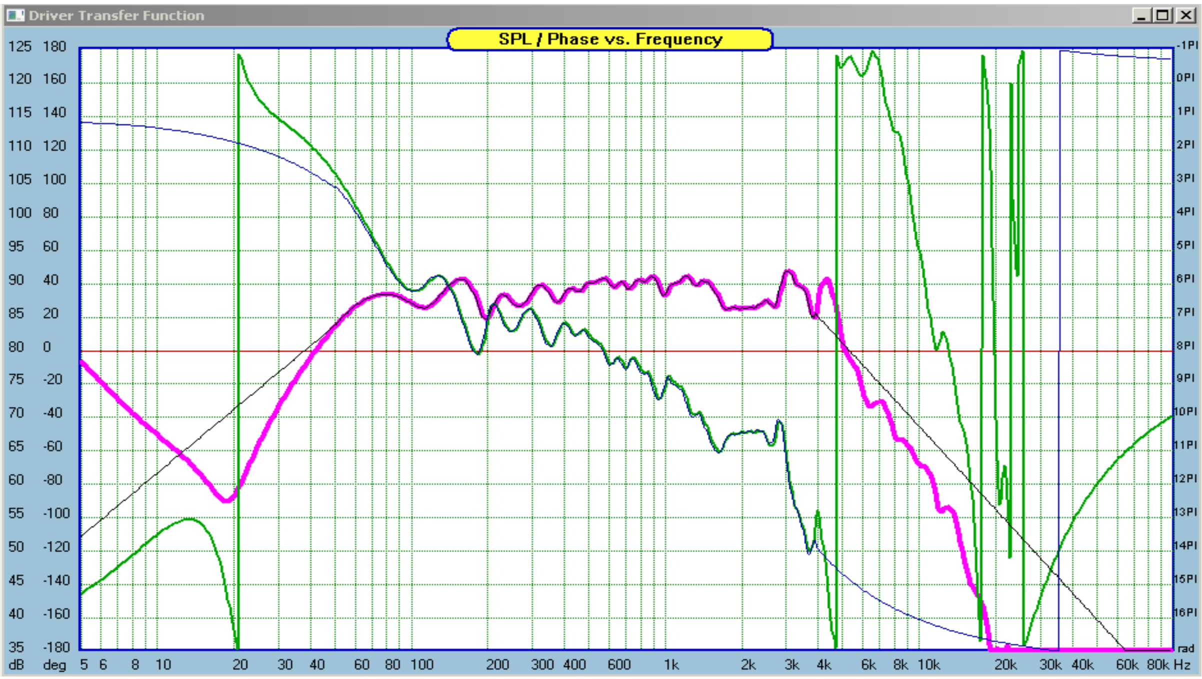

Figure 9. Phase response for the attachment point of 4000Hz.

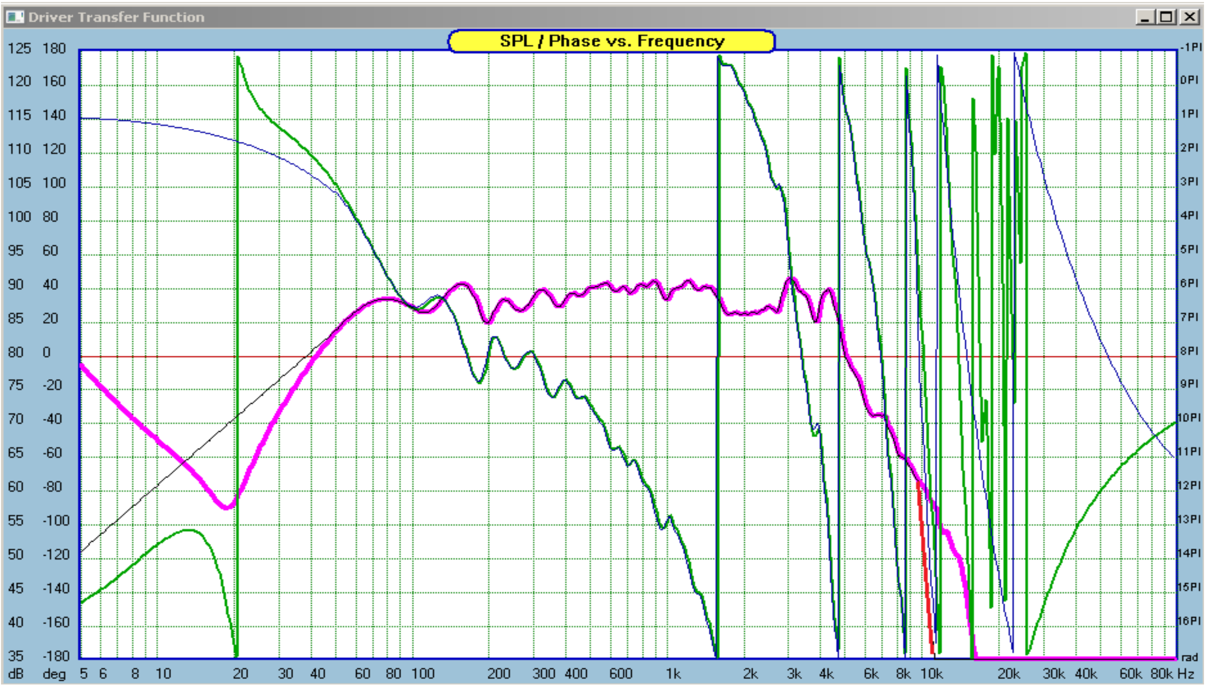

Figure 10. Phase response for the attachment point of 10000Hz.

In both instances, the matching between measured phase and HBT-derived phase is very good. But there are problems in both instances. Please note a completely unrealistic high-frequency slope of 129.45dB/oct (red) at 10kHz on Figure 10. This is plainly wrong, yet, it was recommended for the 10kHz attachment point.

More acceptable phase response is shown on Figure 11 below. It was generated for the following HBT parameters:

| HBT LP Start | HBT HP Start | Bin | Delay | LP Order | HP Order |
|---|---|---|---|---|---|
| 5000 | 50 | 124 | -0.01 | 37.64 | 9.92 |

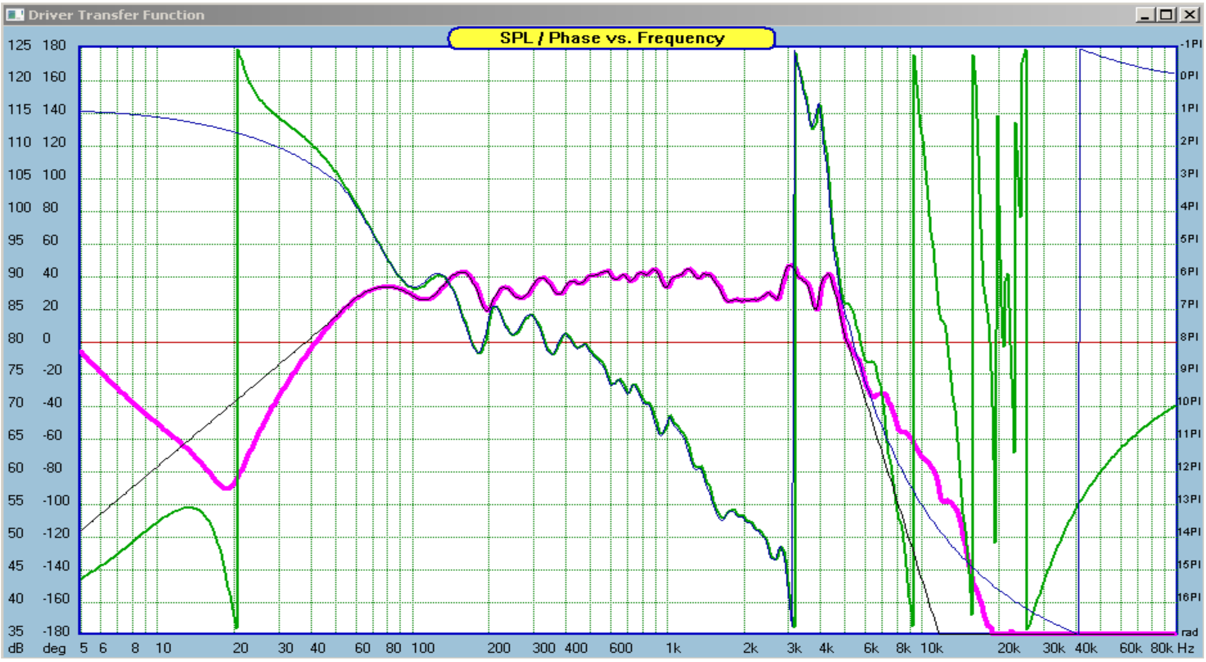


Figure 11. Minimum-phase phase response for Woofer 1.

**Example of Woofer 2**

Once again, the location of the FFT window was arbitrary, so the presented phase response is not a minimum-phase response. The SPL of Woofer 2 is shown on Figure 12 below.

Tabulated responses for the Automated Method for different attachment points are shown below.

| HBT LP Start | HBT HP Start | Bin | Delay | LP Order | HP Order | Error | Bins | Error |
|---|---|---|---|---|---|---|---|---|
| 5000 | 50 | 78 | -0.012 | 24.87 | 15.74 | 2836.7 | 349 | 8.12808 |
| 6000 | 50 | 79 | 0 | 13.67 | 15.76 | 2939.15 | 362 | 8.1192 |
| 7000 | 50 | 79 | -0.002 | 16.47 | 15.76 | 3004.13 | 374 | 8.03243 |
| 8000 | 50 | 78 | -0.01 | 31.84 | 15.76 | 2947.25 | 384 | 7.67513 |
| 9000 | 50 | 77 | 0.004 | 35.21 | 15.73 | 3089.62 | 393 | 7.86163 |
| 9500 | 50 | 77 | -0.002 | 38.34 | 15.72 | 3174.93 | 397 | 7.9973 |

It is interesting to notice, how close (+/-2.6%) the normalized error results are in the second “Error” column. These are all good phase matches, which would be difficult to discriminate visually.

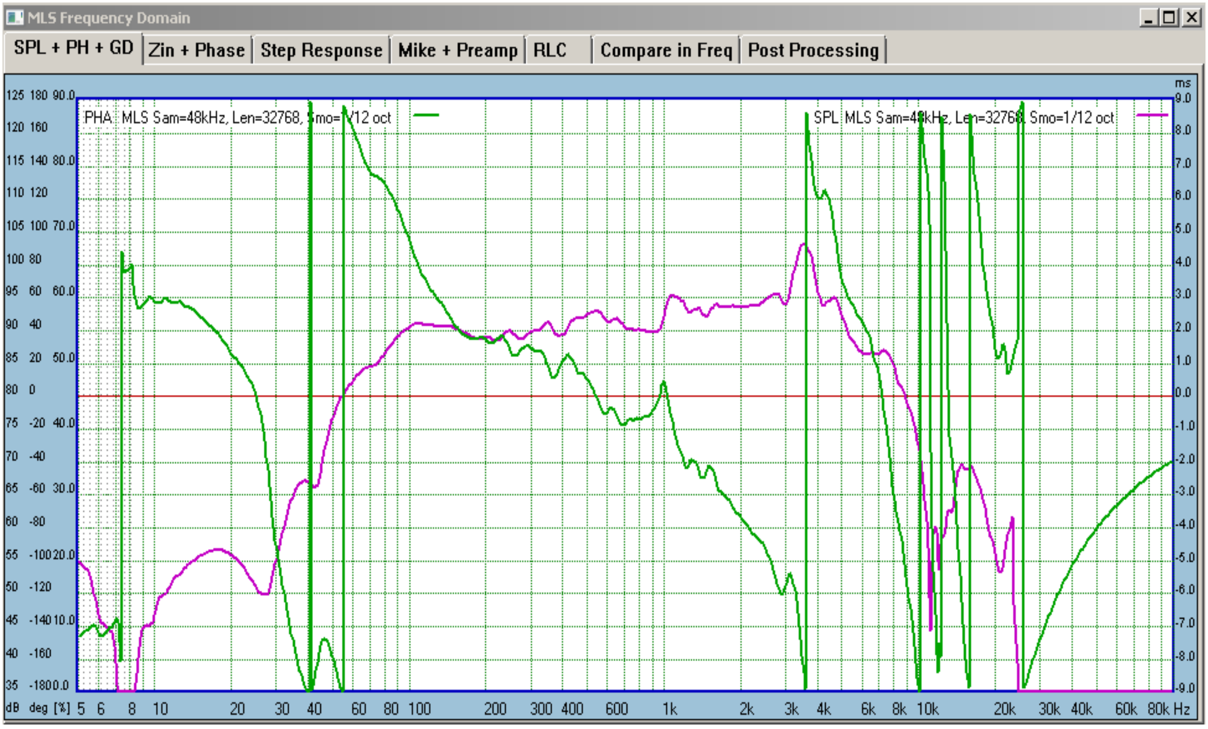


Figure 12. SPL/phase example – Woofer 2 driver.

Not only the Cumulative Error needs to be calculated, but the number of frequency bins also needs to be taken into account. Without this factor, the error results may favour the narrower HBT frequency bandwidth This is illustrated in the tabulated results above. The first "Error" column favours the 5000Hz attachment point. But when the number of frequency bins is accounted for using the "Bins" column, the 8000Hz attachment point would look better in the second "Error" column.

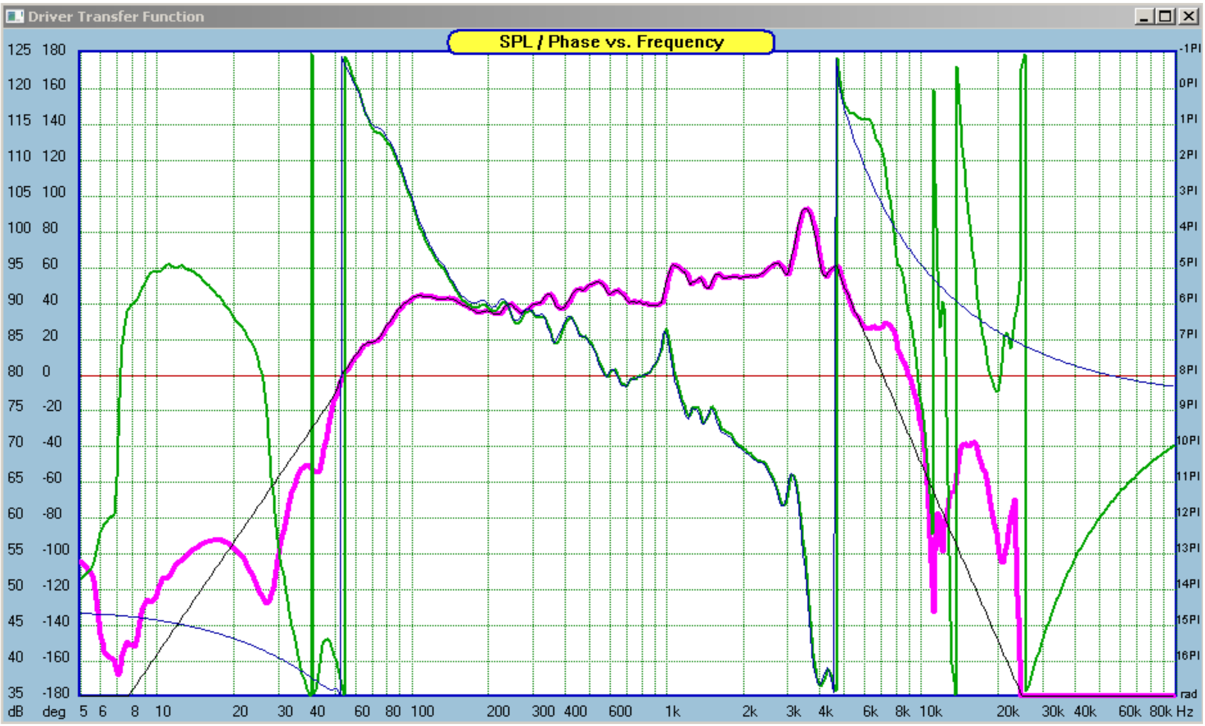


Figure 13. Phase response for the attachment point of 5000Hz.

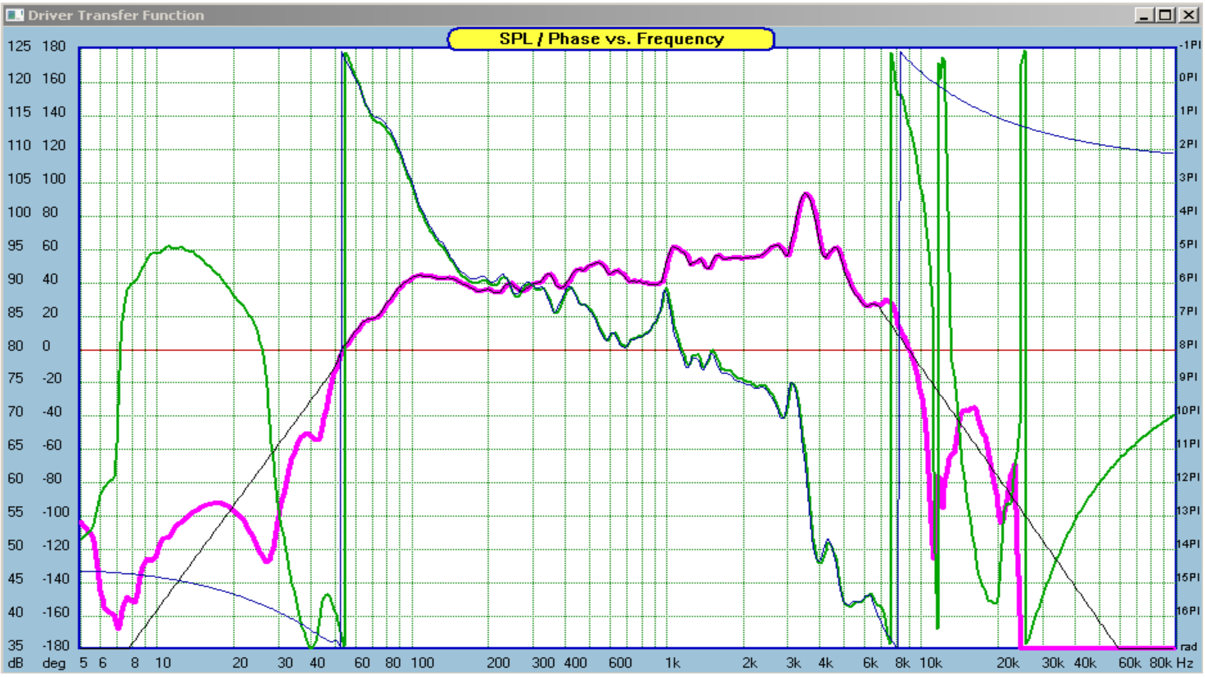

Figure 14. Phase response for the attachment point of 7000Hz.

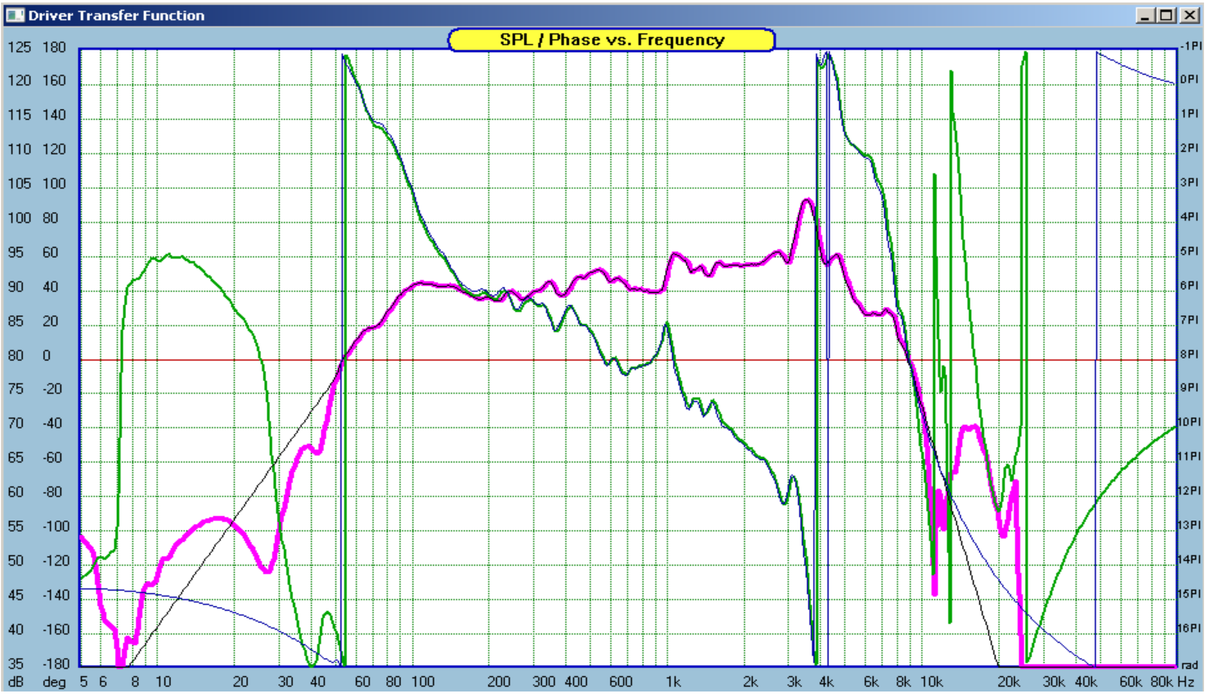

Figure 15. Phase response for the attachment point of 9500Hz.

It is observable, that the Cumulative Error (Error) increases as the HBT frequency range is increased. This is to be expected. This problem can be reduced by taking into account number of frequency bins between the attachment points.

More acceptable phase response is shown on Figure 16 below. It was generated for the following HBT parameters:

| HBT LP Start | HBT HP Start | Bin | Delay | LP Order | HP Order |
|---|---|---|---|---|---|
| 6000 | 100 | 77 | 0 | 27.18 | 15.67 |

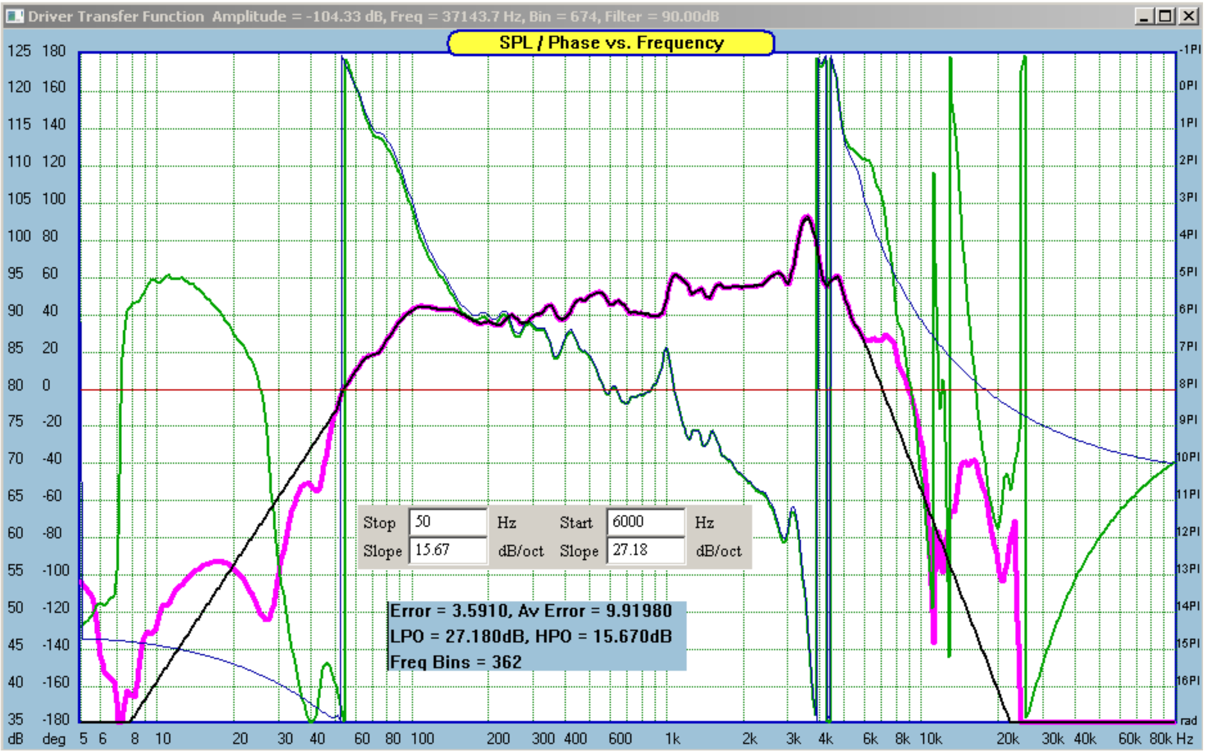


Figure 16. Minimum-phase phase response for Woofer 2.

## Conclusions

The paper presented here highlights some of the issues manifesting themselves during minimum-phase extraction attempts using “manual HBT phase matching” techniques and “automated HBT phase matching techniques”. The processes are difficult.

The manual process relies on visual inspection of the two phase responses and often can be very challenging, as shown of Figure 1, and illustrated by a number of other examples. The automated process delivers Cumulative Error value, and accounts for bandwidth, allowing the user to make informed judgment on the quality of the match.

A number of phase matching examples were presented, where the automated HBT process delivered good indication of matching, and this was supported by presenting the corresponding phase plots. Some were close to the minimum-phase data we were searching for.

Several problems attributed to the selection of the attachment points were also discussed. Selection of attachment points can be challenging as well, as some of them may have to be discarded. Examples presented above would indicate, that testing strategically selected 2-3 attachment points may be sufficient to get good quality data without clogging the picture too much.

The Automated HBT process was specifically designed to include all components of the measurement process. This is because each component can minutely contribute to the final accumulated error result. It is a genuine minimum-phase extraction from measurements.

It was also observed, that extreme cases of phase match can still be recommended by the manual and automated algorithms. The process of eliminating those cases was based on SPL curve, rather than phase responses. For instance, on Figure 3, the phase response (thin blue curve) maybe perfectly acceptable, but the corresponding amplitude response, rolling-off at -8.9dB/oct (thin black curve) shows, that loudspeaker could not have this SPL response. Also, on Figure 10, the phase response is technically acceptable, but the amplitude response rolling off at -126dB/oct is unrealistic.

This is an interesting observation. Here, we are working on determination of the minimum-phase responses, but we are still accounting for corresponding SPL, as the means for discriminating between acceptable and non-acceptable phase responses.

Perhaps a better designed minimum-phase extraction process needs to include the following:

1. Numerical error indication to allow for un-bias selection of phase responses.
2. Process needs to be automated to allow for mathematical precision into the process instead of visual inspection.
3. Optimization for the smallest error needs to be "constrained". In this case, the algorithm would be guided into the area, where the resulting phase response is not of the extreme type, even if the error is not the smallest.
4. As discussed above, the constraint may need to be based on SPL curve, rather than phase response.
5. All components and activities associated with extracting minimum-phase response from measurement need to be included in the phase extraction process.

One issue was evident while examining tabulated responses of all drivers. There was a dependence of the excess phase calculations on the location of the attachment points. This is generally undesirable, because the excess phase is a property of the measurement system distances and remains constant. The excess phase does not change in the fixed system, and the "Bin" and "Delay" values should remain constant. Consequently, one would assume, that changing the attachment points would require changing of the slopes, but the excess phase would remind constant. Tabulated results indeed show, that asymptotic slopes will change, but excess phase will too. The change in excess phase is of moderate size, and often may not present itself as a major problem. This problem will be elaborated upon in Part II of this paper.

Thank you for reading
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